Applied Numerical Mathematics 115 (2017) 99–113

Contents lists available at ScienceDirect

Applied Numerical Mathematics

www.elsevier.com/locate/apnum

Numerical approximation of oscillatory integrals of the linear ship wave theory

Oleg V. Motygin *

Institute for Problems in Mechanical Engineering, Russian Academy of Sciences, V.O., Bol'shoy pr., 61, 199178 St. Petersburg, Russia

A R T I C L E I N F O

Article history:
Received 30 April 2014
Received in revised form 30 November 2016
Accepted 3 January 2017
Available online 7 January 2017

Keywords:
Ship waves
Kelvin wave source
Oscillatory integral
Levin method
Clenshaw–Curtis quadrature
Barycentric Lagrange interpolation

A B S T R A C T

Green's function of the problem describing steady forward motion of bodies in an open ocean in the framework of the linear surface wave theory (the function is often referred to as Kelvin's wave source potential) is considered. Methods for numerical evaluation of the so-called 'single integral' (or, in other words, 'wavelike') term, dominating in the representation of Green's function in the far field, are developed. The difficulty in the numerical evaluation is due to integration over infinite interval of the function containing two differently oscillating factors and the presence of stationary points. This work suggests two methods to approximate the integral. First of them is based on the idea put forward by D. Levin in 1982 — evaluation of the integral is converted to finding a particular slowly oscillating solution of an ordinary differential equation. To overcome well-known numerical instability of Levin's collocation method, an alternative type of collocation is used; it is based on a barycentric Lagrange interpolation with a clustered set of nodes. The second method for evaluation of the wavelike term involves application of the steepest descent method and Clenshaw–Curtis quadrature. The methods are numerically tested and compared.

1. Introduction

The steady motion of a ship in calm water is a classic problem of hydrodynamics, significant for engineering and ship design. As early as in 1887, Lord Kelvin [50] first provided a mathematical description of the wave pattern created by an object moving forward with a constant speed; the familiar V-shaped pattern behind a ship (still being in study, see e.g. [59] and references therein) now bears his name. Starting from Lord Kelvin's work, substantial efforts have been spent to investigation of the ship wave problem and determining the wave resistance. At this, an extensive literature has been produced; comprehensive reviews can be found, e.g. in [56,26]. In particular, much attention has been paid to potential models and linearized statements similar to that considered in the present work.

An important role in the linear theory of ship waves and wave resistance is played by the Kelvin wave source potential, which may also be identified as the Green's function of the Neumann–Kelvin problem. This potential describes a source moving horizontally with a constant velocity in an ideal incompressible fluid having a free surface; fluid's motion is irrotational and steady-state in a coordinate system attached to the source. The wave source potential is fundamental in the theory and applications — for instance, a solution of the problem of the flow past a moving ship is often sought as a dis-

* Fax: +7(812)3214771.
E-mail address: o.v.motygin@gmail.com.

http://dx.doi.org/10.1016/j.apnum.2017.01.003

tribution of the sources on ship's hull. Mathematical view of the method can be found in [26,33] and papers cited therein. More practical treatment, including development of the Fourier–Kochin approach, is suggested in [39,41,58].

Considerable efforts in the analysis of the Green's function have resulted in a vast literature (see [26] and references therein). A number of representations of the Green's function have been derived (see [38]). Generally, the Green's function is divided into three terms: the singular (Rankine) one, whose computation is elementary; the wavelike (referred to as 'single integral') term — oscillating and dominating in the far field; the near-field nonoscillatory term (for historical reasons often referred to as 'double integral term'). Detailed mathematical analysis of the behavior of the near-field and the far-field terms can be found in [9,26]. It is known that the 'double integral term' can be expressed as a single integral, in particular, a number of such representations in terms of the exponential integral function can be found in [49,38]. This allows fast and effective evaluation of the 'double integral term' (see [36,47,20] and references therein).

At the same time, for the wavelike term, the question of evaluation is only partly solved. The main difficulty in computation of the term is due to the presence of two oscillating factors of different nature in the integrand, which can have stationary points, and the infinite interval of integration. Besides, there is a difficult limiting case: it was shown in [53] (more details are given in [9,54]) that the track of the source moving in the free surface is a line of essential singularities. Near the line, the wave elevation oscillates with indefinitely increasing amplitude and indefinitely decreasing wavelength. Of course, in reality, very short ship waves cannot exist due to the influence of surface tension, viscosity and wavebreaking and they are filtered by taking the effects into account (see e.g. [4,40,60]). However, the waves created in the vicinity of the track will be considered below as a good tough test for the algorithms of the present paper.

Most of the existing methods for evaluation of the wavelike term are based on two expansions given in [3]: convergent series and an asymptotic one (completed in [54]). However, the summation of such series is not a trivial computational task, it leads to poor accuracy when the source and the field points are close to the free-surface, the method is vulnerable to severe numerical problems because of the presence of very large in magnitude alternating terms and the criteria to choose one of the expansions is difficult to determine. Details of numerical algorithms can be found in [32,47]. Computation of the wavelike term near the track was discussed in [5] where expansions of the function and numerical algorithms are given.

In the present work, our purpose is to elaborate alternative computation methods to approximate the wavelike term, which should be accurate, fast and applicable for a wide range of parameters. Our consideration is based on recently developed techniques for evaluation of integrals of oscillating functions. The quadrature of oscillatory integrals is a very important computational problem (widely considered as a difficult one) appearing in many applications. The field has enjoyed a recent upsurge of interest and substantive developments with a number of old methods being enhanced and new ones being suggested. Amidst them we mention the numerical steepest descent (see [22,7]), Filon-type (see [13,14,23,43,57]), Levin-type methods (discussed below), Filon–Clenshaw–Curtis quadrature (see [11,8]), 'double exponential formula' for Fourier-type integrals (see [46] and references therein). Reviews of the methods, their comparison and analysis, as well as bibliography notes can be found in [11,24,44,21].

The first of two schemes developed in the present paper is based on the ideas of D. Levin, suggested in [27,28]. The Levin method converts calculation of an oscillatory integral into solving an ordinary differential equation (ODE) whose special, slowly oscillating solution is sought. Typically, the Chebyshev expansion is used as a representation of the solution and the collocation reduces the ODE to a system of linear equations. The Levin method has attracted much attention due to its ability to handle integrals with complicated phase functions, but the collocation has been found to be numerically unstable, leading to an ill-conditioned linear system for a large number of nodes. Analysis and efforts to improve the stability of the Levin collocation method have been made, a number of Levin-type methods have been developed (see [11,10,24,57,29,45,30,31] and references therein). Some progress has been achieved using careful analysis of the linear system and advanced methods of linear algebra such as TSVD (truncated singular value decomposition) and GMRES (generalized minimal residual) method. Our scheme is based on a different type of collocation — instead of the Chebyshev expansion, we use a special form of Lagrange polynomial interpolation which was recently discovered as an effective tool for numerical analysis.

It is to note that polynomial interpolation formulae are widely used in theoretical studies, but not in the numerical practice — in view of their instability. Finding the polynomials involves solution of a Vandermonde linear system of equations, which is exponentially unstable. Runge's phenomenon is also well known: for equispaced interpolation points, small perturbations of the initial data may result in huge changes of the interpolant. However, it has been found (see [48,18,2,52] and references therein) that these problems are avoided when using the Lagrange interpolation in one of the so-called barycentric forms and with the nodes clustered near the ends of the interval of approximation, e.g. at Chebyshev points. A review of the existing results and explanation of the attractive features of the barycentric Lagrange interpolation with a clustered set of nodes can be found in [2,52]; rigorous confirmation of stability is given in [19].

Thus, in our first scheme, following [27,28], we reduce evaluation of the integral in question to solution of an ODE on a finite interval. We prove that the ODE has one bounded solution and define the value of the function at the right end of the interval. Solution's value at the left end of the interval, up to a known factor, coincides with the sought value of the integral. To find the solution of the ODE numerically, we seek it in the form of a barycentric Lagrange interpolating polynomial (alternatively, the representation can include a term arising from asymptotic analysis, intended to absorb 'bad' features of the solution such as sharp peaks). Then we apply collocation of the equation on a set of Chebyshev points. Our numerical experiments show that the constructed numerical algorithm is stable, typically converging steadily to the level of rounding errors with increase of the number of nodes, even in the presence of stationary points (which is considered as complication for Levin and Levin-type methods).