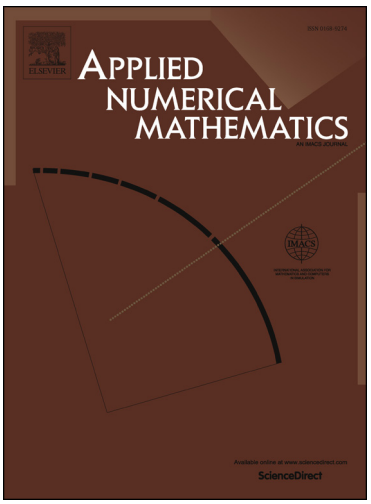
Accepted Manuscript



Analytical and computational methods for a class of nonlinear singular integral equations

Sonia Seyed Allaei, Teresa Diogo, Magda Rebelo

PII: S0168-9274(16)30094-0
DOI: http://dx.doi.org/10.1016/j.apnum.2016.06.001
Reference: APNUM 3043

To appear in: *Applied Numerical Mathematics*

Received date: 16 November 2015
Revised date: 8 May 2016
Accepted date: 7 June 2016

Please cite this article in press as: S.S. Allaei et al., Analytical and computational methods for a class of nonlinear singular integral equations, *Appl. Numer. Math.* (2016), http://dx.doi.org/10.1016/j.apnum.2016.06.001

Analytical and computational methods for a class of nonlinear singular integral equations

Sonia Seyed Allaei[a,b], Teresa Diogo[b,*], Magda Rebelo[c]

[a] *Department of Mathematics, Hong Kong Baptist University, Kowloon Tong, Hong Kong SAR, China*
[b] *Center for Computational and Stochastic Mathematics (CEMAT), Department of Mathematics, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisbon, Portugal*
[c] *Center for Mathematics and Aplications (CMA) and Mathematics Department, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Quinta da Torre, 2829-516 Caparica, Portugal*

Abstract

We consider a general class of nonlinear singular Hammerstein Volterra integral equations. In general, these equations will have kernels containing both an end point and an Abel-type singularity, with exact solutions being typically nonsmooth. Under certain conditions, a uniformly convergent iterative solution is obtained on a small interval near the origin. In this work, two product integration methods are proposed and analysed where the integral over a small initial interval is calculated analytically, allowing the optimal convergence rates to be achieved. This is illustrated by some numerical examples.

Keywords:
Nonlinear Volterra integral equation, weakly singular kernel, Hammerstein equations, iterative solution, product integration method.
2000 MSC: 65R20, 45J05

1. Introduction

This work is concerned with analytical and numerical results for nonlinear Volterra integral equations of the form

$$y(t) = f(t) - \int_0^t \frac{s^\beta}{(t-s)^\alpha} g(y(s))ds, \quad t > 0, \tag{1}$$

where α, β are positive real constants.

We start with a review on papers dedicated to applications which give rise to VIEs of the above form. In the study of many nonlinear problems in heat conduction, boundary-layer heat transfer, chemical kinetics and superfluidity, we are often led to singular Volterra integral equations of the type [17]

$$y^m(t) = H(t) + \gamma\, t^{-\mu} \int_0^t \frac{G(y(s), y'(s), s)}{(t^\beta - s^\beta)^\alpha} ds, \quad z > 0, \tag{2}$$

*Corresponding author
Email addresses: soniase@gmail.com (Sonia Seyed Allaei), tdiogo@math.ist.utl.pt (Teresa Diogo), msjr@fct.unl.pt (Magda Rebelo)