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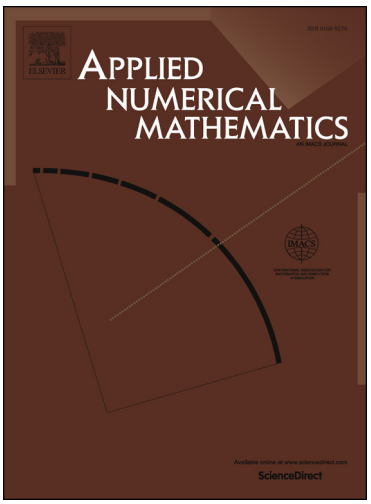

A Novel Computational Approach to Singular Free Boundary Problems in Ordinary Differential Equations

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A Novel Computational Approach to Singular Free Boundary Problems in Ordinary Differential Equations

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Abstract

We study the numerical solution of a singular free boundary problem for a second order nonlinear ordinary differential equation, where the differential operator is the degenerate m-Laplacian. A typical difficulty arising in free boundary problems is that the analytical solution may become non-smooth at one boundary or at both boundaries of the interval of integration. A numerical method proposed in [18] consists of two steps. First, a smoothing variable transformation is applied to the analytical problem in order to improve the smoothness of its solution. Then, the problem is discretized by means of a finite difference scheme.

In the present paper, we consider an alternative numerical approach. We first transform the original problem into a special parameter dependent problem sometimes referred to as an 'eigenvalue problem'. By applying a smoothing variable transformation to the resulting equation, we obtain a new problem whose solution is smoother, and so the open domain MATLAB collocation code `bvpsuite` [16] can be successfully applied for its numerical approximation.

Keywords: Degenerate Laplacian, singular free boundary problem, smoothing variable substitution, collocation methods.
2000 MSC: 65L05,34B16

1. Introduction

Many mathematical models in physics and mechanics can be formulated as the following free boundary problem (FBP): Find a real number $M > 0$ and a positive solution $C^1[0, M] \cap C^2(0, M)$ of the equation

$$\left(|y'(x)|^{m-2}\, y'(x)\right)' + \frac{N-1}{x}\, |y'(x)|^{m-2}\, y'(x) + f(y(x)) = 0, \quad 0 < x < M, \tag{1}$$

such that y satisfies the boundary conditions

$$y'(0) = 0, \quad y(M) = y'(M) = 0. \tag{2}$$

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