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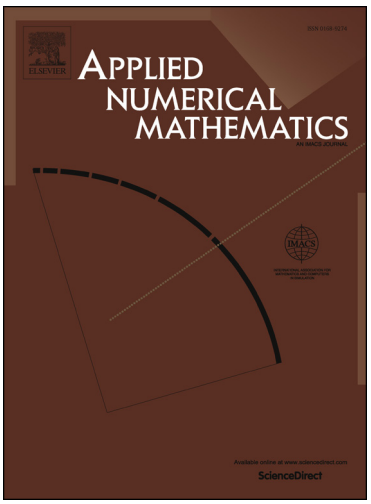

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Numerical solution for diffusion equations with distributed order in time using a Chebyshev collocation method

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Abstract

In this work we present a new numerical method for the solution of the distributed order time-fractional diffusion equation. The method is based on the approximation of the solution by a double Chebyshev truncated series, and the subsequent collocation of the resulting discretised system of equations at suitable collocation points. An error analysis is provided and a comparison with other methods used in the solution of this type of equation is also performed.

Keywords: Fractional differential equation, Caputo derivative, Diffusion equation, Chebyshev polynomials, Spectral methods, Distributed order equation.
2000 MSC: 26A33, 41A50

1. Introduction

In 1827, the botanist Robert Brown observed the intriguing jittering movement of small particles such as pollen grains, when these were immersed in water. Nowadays it is well known that this motion is caused by the rapid movement of water molecules, and insight into this problem was provided by Albert Einstein in 1905, in his work regarding Brownian motion, entitled "On the motion, required by the molecular-kinetic theory of heat, of particles suspended in fluids at rest." [22]. His work served as a definitive confirmation that atoms and molecules actually exist. Although molecules are too small to be seen directly, their presence can be inferred from their visible effect on larger grains (such as the pollen grains). In his doctoral dissertation, Einstein developed a statistical molecular theory of liquids and in his subsequent paper he took the view that Brownian motion could be explained in terms of a type of stochastic process called a "random walk" (it is worth mentioning that Louis Bachelier, a student of Henri Poincaré, developed a theory of Brownian motion in his 1900 thesis [1] regarding stock market fluctuation [39]).

The random walk theory was popularised by Karl Pearson in his letter to Nature (1905) [55], where he proposed the following problem: a man starts from a point O and walks l yards in a

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