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Optical flow with fractional order regularization: Variational model and solution method

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A B S T R A C T

An optical flow variational model is proposed for a sequence of images defined on a domain in $\mathbb{R}^2$. We introduce a regularization term given by the L^1 norm of a fractional differential operator. To solve the minimization problem we apply the split Bregman method. Extensive experimental results, with performance evaluation, are presented to demonstrate the effectiveness of the new model and method and to show that our algorithm performs favorably in comparison to another existing method. We also discuss the influence of the order α of the fractional operator in the estimation of the optical flow, for $0 \le \alpha \le 2$. We observe that the values of α for which the method performs better depend on the geometry and texture complexity of the image. Some extensions of our algorithm are also discussed.

1. Introduction

Optical flow is a tool for detecting and analyzing motion in a sequence of images. The underlying idea is to depict the displacement of patterns in the image sequence as a vector field, named the optical flow vector field, generating the corresponding displacement function. In their seminal paper, Horn and Schunck [12] suggested a variational method for the computation of the optical flow vector field. In this approach the goal is to minimize an energy functional consisting of a similarity term (or data term) and a regularity term:

$$\underset{u \in \mathcal{H}}{\operatorname{argmin}}\, E(\mathbf{u}) = \underset{\mathbf{u} \in \mathcal{H}}{\operatorname{argmin}}(\mathcal{R}(\mathbf{u}) + \mathcal{S}(\mathbf{u})).$$

The space $\mathcal{H}$ denotes an admissible space of vector fields, $\mathcal{R}$ denotes the regularity term for the vector field $\mathbf{u}$, and $\mathcal{S}$ denotes the similarity term that depends on the data image sequence. In particular the functional is of the form [12]

$$E(\mathbf{u}) = \beta^2 \int_{\Omega} (|\nabla u_1|^2 + |\nabla u_2|^2) d\Omega + \int_{\Omega} (I_1(\mathbf{x} + \mathbf{u}(\mathbf{x})) - I_0(\mathbf{x}))^2 d\Omega. \tag{1}$$

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Here, I_0 and I_1 is the image pair, $\mathbf{u} = (u_1(\mathbf{x}), u_2(\mathbf{x}))^T$ is the two-dimensional displacement field and β is a fixed parameter. The first term (regularization term) penalizes high variations in $\mathbf{u}$ to obtain smooth displacement fields. The second term (data term) is also known as the optical flow constraint. It assumes, that the intensity values of $I_0(\mathbf{x})$ do not change during its motion to $I_1(\mathbf{x}+\mathbf{u}(\mathbf{x}))$. Horn and Schunck [12] observed that β^2 plays a significant role only for areas where the brightness gradient is small, preventing haphazard adjustments to the estimated flow velocity. Disadvantages of this model consist of not preserving discontinuities in the flow field and of not handling outliers efficiently. To overcome the difficulties presented by the Horn–Schunck functional, several extensions and improvements have been developed [21].

In [22] the optical flow model proposed consists in considering an L^1 norm in the regularizing term and the similarity term is substantially changed by introducing an auxiliary variable $\mathbf{v}$. The process is a result of first changing the quadratic factors that appeared in the classical method (1), obtaining an energy functional which is the sum of the total variation of $\mathbf{u}$ and an L^1 term:

$$E(\mathbf{u}) = \int_\Omega |\nabla \mathbf{u}| d\Omega + \lambda \int_\Omega |\rho(\mathbf{u})| d\Omega, \tag{2}$$

where $|\nabla \mathbf{u}| = |\nabla u_1| + |\nabla u_2|$ and the image residual denoted by $\rho(\mathbf{u})$ (we omitted the explicit dependency on $\mathbf{u}^0$ and $\mathbf{x}$) is given by

$$\rho(\mathbf{u}) = \nabla I_1(\mathbf{x}+\mathbf{u}^{\mathbf{0}}).(\mathbf{u}-\mathbf{u}^0) + I_1(\mathbf{x}+\mathbf{u}^0) - I_0(\mathbf{x}). \tag{3}$$

The vector $\mathbf{u}^0$ is a given disparity map and the functional was obtained for a fixed $\mathbf{u}^0$ and using the linear approximation for $I_1(\mathbf{x}+\mathbf{u})$ near $\mathbf{x}+\mathbf{u}^0$.

Secondly, a convex relaxation term is introduced [22] in order to minimize this energy functional efficiently obtaining

$$E_\theta(\mathbf{u},\mathbf{v}) = \int_\Omega \left\{ |\nabla \mathbf{u}| + \frac{1}{2\theta}|\mathbf{u}-\mathbf{v}|^2 + \lambda|\rho(\mathbf{v})| \right\} d\Omega, \tag{4}$$

where θ is a small constant, such that $\mathbf{v}$ is a close approximation of $\mathbf{u}$. Setting θ very small forces the minimum of E_θ to occur when $\mathbf{u}$ and $\mathbf{v}$ are nearly equal, reducing the energy (4) to the original energy (2).

Many approaches for optical flow computation replace the nonlinearity intensity profile $I_1(\mathbf{x}+\mathbf{u})$ by a first Taylor approximation to linearize the problem locally as in the case presented above. Since such approximations are only valid for small motions, in the presence of large displacements, the method fails when the gradient of the image is not smooth enough. This means that additional techniques are required to determine the optical flow correctly. Therefore an iterative warping is applied in the implementation to compensate for image nonlinearities. A multiscale strategy is also included to allow disparities between the images.

In this work we propose an optical flow model for a sequence of images defined on a domain in $\mathbb{R}^2$ which consists of a modification of the model introduced in [22], by considering for the regularization term the L^1 norm of a fractional derivative operator [9]. The numerical method developed to solve the minimization problem involves a multiscale strategy [15] and the split Bregman method described in [11]. The effectiveness of the new model and numerical approach is shown by presenting experimental results that use the test sequences available in the Middlebury benchmark database designed by [2]. We also compare its performance with other existing numerical method.

In the next section we present the variational method and in Section 3 we describe the numerical approach which includes the split Bregman method, Euler Lagrange equations, a shrinkage operator, a thresholding operator and finite differences. In Section 4 several experiments are shown and we end with some conclusions and general comments in Section 5.

2. Problem formulation

We propose a generalized method that involves fractional derivatives in the regularization term. Recently fractional derivatives have been brought to the field of image processing and fractional differentiation based methods have been demonstrating advantages over already existing methods, see for instance [8,9,17,23]. There are different definitions of fractional derivatives. Until now, in the subject of image processing, the fractional Riemann–Liouville derivative has been widely adopted and is the one used herein. Different reasons could be behind this choice. The fractional Riemann Liouville derivative can be defined for less regular functions, it is naturally related with diffusive processes and can be easily discretized through the standard Grünwald–Letnikov approximation. Let us introduce the definition of fractional Riemann–Liouville derivative.

The left Riemann–Liouville derivative of order α, for a scalar function u, is defined by

$$D_-^\alpha u(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_a^t u(\tau)(t-\tau)^{m-\alpha-1} d\tau, \quad m-1 < \alpha < m, \tag{5}$$

for $a \le t \le b$, where m is a positive integer and Γ denotes the Gamma function