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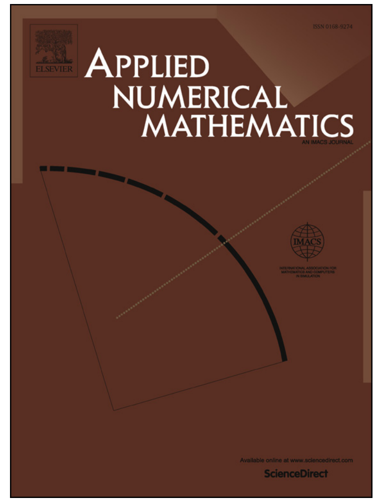

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Error estimates of a high order numerical method for solving linear fractional differential equations

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Abstract

In this paper, we first introduce an alternative proof of the error estimates of the numerical methods for solving linear fractional differential equations proposed in Diethelm [6] where a first-degree compound quadrature formula was used to approximate the Hadamard finite-part integral and the convergence order of the proposed numerical method is $O(\Delta t^{2-\alpha}), 0 < \alpha < 1$, where α is the order of the fractional derivative and Δt is the step size. We then use a similar idea to prove the error estimates of the high order numerical method for solving linear fractional differential equations proposed in Yan et al. [37], where a second-degree compound quadrature formula was used to approximate the Hadamard finite-part integral and we show that the convergence order of the numerical method is $O(\Delta t^{3-\alpha}), 0 < \alpha < 1$. Nnumerical examples are given to show that the numerical results are consistent with the theoretical results.

Keywords:
Fractional differential equations, fractional derivative, error estimates
AMS Subject Classification: 65M12; 65M06; 65M70;35S10

1. Introduction

In this paper, we consider numerical methods for solving the following linear fractional differential equation

$$ {}_0^C D_t^{\alpha} x(t) = \beta x(t) + f(t), \quad t \in [0,1], \tag{1} $$

$$ x(0) = x^0, \tag{2} $$

where $0 < \alpha < 1$ and $\beta < 0$, $x^0 \in \mathbb{R}$ denotes the initial value, f is a given function on the interval $[0,1]$ and ${}_0^C D_t^{\alpha} x(t)$ denotes the Caputo fractional order derivative.

Diethelm [6] introduced a numerical method for solving (1)-(2) by approximating the Hadamard finite-part integral with the first-degree compound quadrature formula and proved that the convergence order is $O(\Delta t^{2-\alpha})$, where Δt is the step size. Ford et al. [15] used the similar method to consider the time discretization of the following time-fractional partial differential equation

$$ {}_0^C D_t^{\alpha} u(t,x) - \Delta u(t,x) = f(t,x), \quad t \in [0,T], \ x \in \Omega, \tag{3} $$

$$ u(0,x) = 0, \quad x \in \Omega, \tag{4} $$

$$ u(t,x) = 0, \quad t \in [0,T], \ x \in \partial\Omega, \tag{5} $$

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