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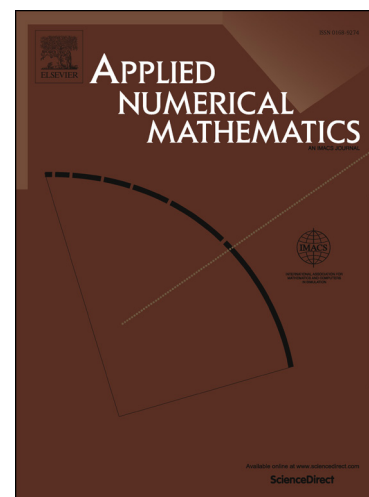
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Error estimates of a high order numerical method for solving linear fractional differential equations

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Abstract

In this paper, we first introduce an alternative proof of the error estimates of the numerical methods for solving linear fractional differential equations proposed in Diethelm [6] where a first-degree compound quadrature formula was used to approximate the Hadamard finite-part integral and the convergence order of the proposed numerical method is $O(\Delta t^{2-\alpha})$, $0 < \alpha < 1$, where α is the order of the fractional derivative and Δt is the step size. We then use a similar idea to prove the error estimates of the high order numerical method for solving linear fractional differential equations proposed in Yan et al. [37], where a second-degree compound quadrature formula was used to approximate the Hadamard finite-part integral and we show that the convergence order of the numerical method is $O(\Delta t^{3-\alpha})$, $0 < \alpha < 1$. Numerical examples are given to show that the numerical results are consistent with the theoretical results.

Keywords:

Fractional differential equations, fractional derivative, error estimates

AMS Subject Classification: 65M12; 65M06; 65M70; 35S10

1. Introduction

In this paper, we consider numerical methods for solving the following linear fractional differential equation

$${}_0^C D_t^\alpha x(t) = \beta x(t) + f(t), \quad t \in [0, 1], \quad (1)$$

$$x(0) = x^0, \quad (2)$$

where $0 < \alpha < 1$ and $\beta < 0$, $x^0 \in \mathbb{R}$ denotes the initial value, f is a given function on the interval $[0, 1]$ and ${}_0^C D_t^\alpha x(t)$ denotes the Caputo fractional order derivative.

Diethelm [6] introduced a numerical method for solving (1)-(2) by approximating the Hadamard finite-part integral with the first-degree compound quadrature formula and proved that the convergence order is $O(\Delta t^{2-\alpha})$, where Δt is the step size. Ford et al. [15] used the similar method to consider the time discretization of the following time-fractional partial differential equation

$${}_0^C D_t^\alpha u(t, x) - \Delta u(t, x) = f(t, x), \quad t \in [0, T], \quad x \in \Omega, \quad (3)$$

$$u(0, x) = 0, \quad x \in \Omega, \quad (4)$$

$$u(t, x) = 0, \quad t \in [0, T], \quad x \in \partial\Omega, \quad (5)$$

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