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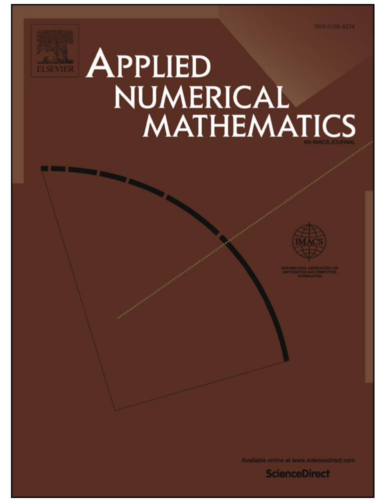

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Approximating the leading singular triplets of a large matrix function

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Abstract

Given a large square matrix A and a sufficiently regular function f so that $f(A)$ is well defined, we are interested in the approximation of the leading singular values and corresponding left and right singular vectors of $f(A)$, and in particular in the approximation of $\|f(A)\|$, where $\|\cdot\|$ is the matrix norm induced by the Euclidean vector norm. Since neither $f(A)$ nor $f(A)\mathbf{v}$ can be computed exactly, we introduce a new *inexact* Golub-Kahan-Lanczos bidiagonalization procedure, where the inexactness is related to the inaccuracy of the operations $f(A)\mathbf{v}$, $f(A)^*\mathbf{v}$. Particular outer and inner stopping criteria are devised so as to cope with the lack of a true residual. Numerical experiments with the new algorithm on typical application problems are reported.

1. Introduction

Given a large $n \times n$ complex matrix A and a sufficiently regular function f so that $f(A)$ is well defined, we are interested in approximating the largest singular values and corresponding left and right singular vectors of the matrix function $f(A)$; we shall refer to these three quantities as *triplet*. This computation will also give an approximation to the 2-norm $\|f(A)\|$, where $\|\cdot\|$ is the matrix norm induced by the Euclidean vector norm, and it is defined as

$$\|f(A)\| = \max_{0 \neq \mathbf{x} \in \mathbb{C}^n} \frac{\|f(A)\mathbf{x}\|}{\|\mathbf{x}\|}. \tag{1}$$

In our presentation we will chiefly discuss this norm approximation because of its interest in applications. However, we shall keep in mind that the considered procedure allows us to also determine singular triplets $(\sigma, \mathbf{u}, \mathbf{v})$, and that a group of singular triplets can be determined simultaneously.

The problem of approximating the norm of a matrix function arises in the solution of stiff linear initial value problems [15],[33], in the evaluation of derivatives and perturbations of matrix functions, which arise for instance in