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Reconstruction of 3D scattered data via radial basis functions by efficient and robust techniques

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ABSTRACT

We propose new algorithms to overcome two of the most constraining limitations of surface reconstruction methods in use. In particular, we focus on the large amount of data characterizing standard acquisitions by scanner and the noise intrinsically introduced by measurements. The first algorithm represents an adaptive multi-level interpolating approach, based on an implicit surface representation via radial basis functions. The second algorithm is based on a least-squares approximation to filter noisy data. The third approach combines the two algorithms to merge the correspondent improvements. An extensive numerical validation is performed to check the performances of the proposed techniques.

1. Introduction

Surface reconstruction consists in retrieving a virtual model starting from a set of scattered three-dimensional data lying on the surface of an object of interest. This issue characterizes many present-day technological applications, from medicine to facial recognition systems, or to the fine arts. The virtual model can be eventually used for visualization or diagnostic purposes, for industrial reverse engineering [5] or to generate a polygonal mesh aimed to scientific computing.

This paper provides new algorithms which tackle two of the most constraining limitations of surface reconstruction, i.e., the size of the data sampling typical, for instance, of standard acquisitions by scanner and the noise intrinsically introduced by measurements. The proposed algorithms are based on an implicit representation of the surface of interest via Radial Basis Functions (RBF) and improve an algorithm previously proposed in [18]. In more detail, we modify the procedure proposed there, relying on a multi-level approach to merge local and global supported RBF. Several advantages are guaranteed by a multi-level reconstruction, we mention, in particular, the sparsity of the matrix, a highly accuracy of recovered surfaces with details at different scales or non-uniformly distributed data, a good quality polygonization of the surface (see Fig. 1 for an example, from the Stanford 3D scanning repository [24]). Nevertheless, the multi-level reconstruction still exhibits a local redundancy in the data and a marked sensitivity to noise. These drawbacks justify the proposal of the new algorithms in this paper.

The first algorithm is suited to efficiently deal with large data set, by avoiding any redundancy in the selection of the data used for surface reconstruction. In short, it may be considered as an adaptive version of the multi-level approach and provides an accurate approximation of a surface Σ resorting to a limited number of points. We complete the multi-level algorithm with an error indicator to refine the cloud of points where Σ exhibits the most complex behavior.

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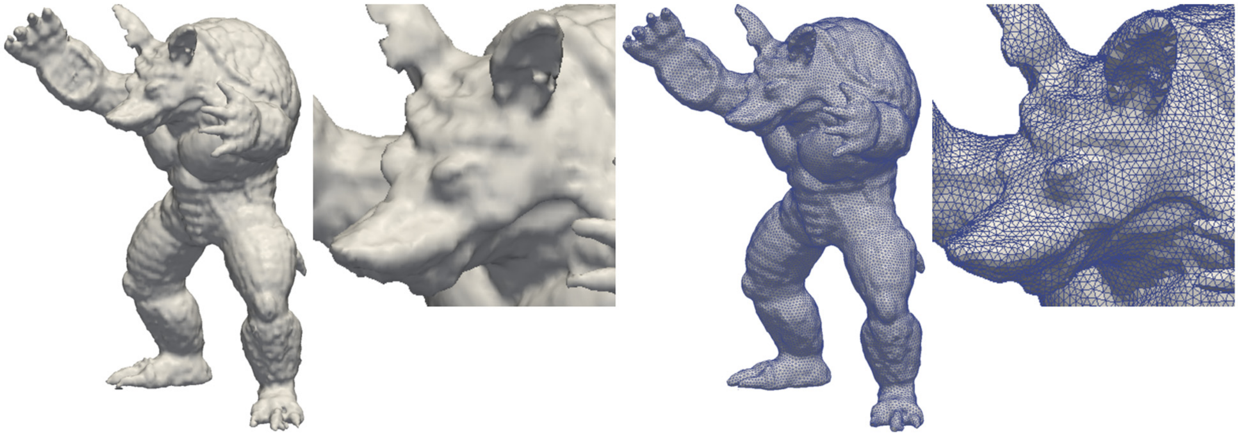

Fig. 1. The Stanford armadillo model: surface reconstruction (left) and corresponding polygonization (right).

The second variant is proposed to handle the reconstruction of noisy data. For this purpose, we advocate a least-squares technique which is properly regularized via a ridge regression approach [10].

The third algorithm provides an appropriate merging of the adaptive and of the least-squares procedures to tackle large sparse and noisy data sets.

The paper is organized as follows. In Section 2, we furnish the multi-level interpolation method presented in [18], after a short introduction on RBF interpolation for scattered data. A detailed numerical check is performed to verify the actual advantages led by this approach. Section 3 details the first new procedure, i.e., the adaptive algorithm. The associated numerical investigation concerns standard data sets as well as data from medical measurements. This last check turns out to be very challenging due to complexity of the considered geometries. In Section 4, we introduce the least-squares counterpart of the multi-level algorithm and we numerically assess the robustness of such a scheme. Finally, in Section 5 we combine the adaptive approach with the least-squares procedure, to merge the advantages of the two algorithms. Some conclusions are drawn in the last section and possible developments of the work are provided.

2. Interpolation of scattered data via RBF

Let us consider a cloud $X = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$ of N scattered points describing an unknown surface $\Sigma \subset \mathbb{R}^3$. We assume that points in X are endowed with unit inward normals, collected in $\Gamma = \{\mathbf{n}_1, \ldots, \mathbf{n}_N\}$, thus defining an orientation on Σ. We aim at reconstructing Σ by exploiting the data in X and Γ. This issue can be generally formulated as the multivariate interpolation problem: given $X = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\} \subset \mathbb{R}^3$ and $S = \{f_1, f_2, \ldots, f_N\} \subset \mathbb{R}$, with $f_j = f(\mathbf{x}_j)$ for $1 \le j \le N$, find a continuous function $F : \mathbb{R}^3 \to \mathbb{R}$ such that

$$F(\mathbf{x}_j) = f_j \quad \text{for } 1 \le j \le N. \tag{1}$$

The scalar function F represents the unknown we are interested in, where we assume Σ to coincide with the zero level set $\{\mathbf{x} \in \mathbb{R}^3 : F(\mathbf{x}) = 0\}$ of F, according to an implicit representation of the surface. This simply leads us to select in (1) $f_j = 0$, for $1 \le j \le N$.

The multivariate interpolation is a problem less standard with respect to the well-established univariate interpolation [21]. In more detail, according to the Mairhuber–Curtis theorem, it is not *a priori* guaranteed the existence of a multivariate polynomial interpolating an arbitrary set of data in $\mathbb{R}^3$ [29].

In the sequel we resort to a multivariate interpolation based on RBF. This type of interpolation is not covered by the Mairhuber–Curtis theorem. In such a case, problem (1) is well-posed under reasonable assumptions on the selected radial function and on the set X [29].

In the literature, we may distinguish between *globally* or *compactly* supported RBF (for an interesting comparison, we refer to [16]). Globally supported RBF provide good-quality reconstructions even for non uniformly distributed or incomplete data. At the same time, the global support of F associates a full interpolation matrix with conditions (1), thus making the evaluation of F at a generic point $\mathbf{x} \in \mathbb{R}^3$ computationally expensive, especially when the cardinality of X becomes large [5,25,12,9,26,4]. On the contrary, compactly supported RBF are characterized by a sparse interpolation matrix. This significantly reduces the computational effort to evaluate $F(\mathbf{x})$, while ensuring that data perturbations have only a local effect on the reconstructed surface. Nevertheless, compactly supported RBF are not suited to deal with incomplete data. Moreover, the compact support of F, which is in general confined to a thin layer around Σ, may be problematic during the polygonization of the surface. A polygonization step smaller than the thickness of this layer is necessarily required.

In the sequel, we adopt the approach proposed in [18] which merges the computational efficiency of compactly supported RBF with the robustness of a globally supported RBF approximation. The interpolating function F in (1) is chosen as