



Extension of universal cycles for globally identifying colorings of cycles



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ABSTRACT

In 1998, Karpovsky, Chakrabarty and Levitin introduced identifying codes to model fault diagnosis in multiprocessor systems (Karpovsky et al., 1998). In these codes, each vertex is identified by the vertices belonging to the code in its neighborhood. There exists a coloring variant as follows: a globally identifying coloring of a graph is a coloring such that each vertex is identified by the colors in its neighborhood. We aim at finding the maximum length of a cycle with such a coloring, given a fixed number of colors we can use. Parreau (2012) used Jackson's work (Jackson, 1993) on universal cycles to give a lower bound of this length. In this article, we will adapt what Jackson did, to improve this result.

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1. Introduction

In this article, we will build a globally identifying coloring of a cycle. This coloring is a variant of identifying codes.

Definition 1 (*Identifying Code*). An identifying code C of a graph $G = (V, E)$ is a subset of V such that:

- (1) $\forall x \in V, C \cap N[x] \neq \emptyset$
- (2) $\forall x, y \in V, x \neq y, C \cap N[x] \neq C \cap N[y]$

where $N[x]$ is the closed neighborhood of x , that is to say the neighbors of x plus x itself.

Definition 2. The minimum size of an identifying code of a graph G is denoted by $\gamma_{id}(G)$.

These codes were introduced by Karpovsky, Chakrabarty and Levitin [10] to solve the problem of fault diagnosis of multiprocessor systems. The processors can be seen as the nodes of a graph, while the edges are the communication links between processors. Each processor may run a software routine which can detect an error in the processor itself or one of its neighborhood. The subset of processors which will run this routine must be a identifying code of the graph. The condition 1 of Definition 1 expresses that if an error occurs, we want to detect it. The condition 2 allows to localize where the problem happened; the error is detected by some processors running the routine, and this subset of processors is unique. The aim is to have the minimum number of processors running the routine, that is to minimize $|C|$.

Remark that these codes can only exist if there are no twin vertices in the graph. Indeed, if there are two vertices x and y such that $N[x] = N[y]$, then it is impossible to satisfy the condition 2 of Definition 1. Therefore we will only consider graph without twin vertices.

We know some bounds of $\gamma_{id}(G)$.

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Fig. 1. An identifying code and a globally identifying coloring of a P_3 .

Proposition 1 ([10]). For a graph G with n vertices, $\gamma_{id}(G) \geq \log_2(n + 1)$.

There exist several variants of identifying codes. For instance we could expect to identify subsets of vertices rather than vertices [3,6,7]. We could also require to identify vertices at distance at most r : these are called r -identifying codes [1,4,5]. In this paper we focus on an extension of identifying codes to colorings.

Definition 3 (Coloring). A coloring c of a graph $G = (V, E)$ is a function from the set of vertices V to a set of colors, represented by $\mathbb{N}$.

Given a subset of vertices $S \subseteq V$, $c(S) = \{c(x) : x \in S\}$ is the set of the colors of vertices in S .

Definition 4 (Globally Identifying Coloring). A globally identifying coloring of a graph $G = (V, E)$ is a coloring c of this graph, such that:

$$\forall x, y \in V, x \neq y, \quad c(N[x]) \neq c(N[y]).$$

Remark that the coloration does not need to be proper. That is to say there can be two neighbors with the same color.

Definition 5. The minimum number of colors of a globally identifying coloring of a graph G is denoted by $\chi_{id}(G)$.

There exist links between identifying codes and globally identifying colorings.

Theorem 1. For a graph G , $\chi_{id}(G) \leq \gamma_{id}(G) + 1$.

Proof. Let C be an identifying code of size of G . We can build a globally identifying coloring of G as follows: all the vertices in C have a unique color in $\{1, \dots, |C|\}$; the others have the color 0. This coloring uses $|C| + 1$ colors and is indeed a globally identifying coloring. Let x and y be two vertices of G . The code C is identifying so $C \cap N[x] \neq C \cap N[y]$. Thus there exists a vertex z in the symmetric difference of $C \cap N[x]$ and $C \cap N[y]$, that is z is in the code and in the neighborhood of only one vertex among x and y . The vertex z has a unique color, so $c(N[x]) \neq c(N[y])$ as $c(z)$ is in the symmetric difference. $\square$

This bound is tight for some graphs. For example, as we can see in Fig. 1, $\gamma_{id}(P_3) = 2$ and $\chi_{id}(P_3) = 3$.

These globally identifying colorings also exist in a local variant.

Definition 6 (Locally Identifying Coloring). A locally identifying coloring of a graph $G = (V, E)$ is a coloring c of this graph, such that:

$$\forall (x, y) \in E, \quad c(N[x]) \neq c(N[y]).$$

In a locally identifying coloring, we only need to identify adjacent vertices.

Definition 7. The minimum number of colors of a locally identifying coloring of a graph G is denoted by $\chi_{lid}(G)$.

Cycles have been investigated for identifying codes [4] and r -identifying codes [1,9], as well as for locally identifying colorings [2].

Theorem 2 ([2]). Let $n \geq 4$ be an integer, and C_n be the cycle of length n . Then:

- $\chi_{lid}(C_n) = 3$ if $n \equiv 0 \pmod{4}$
- $\chi_{lid}(C_n) = 5$ if $n = 5$ or $n = 7$
- $\chi_{lid}(C_n) = 4$ else.

Theorem 2 gives us the minimum number of colors we need to build a locally identifying coloring of a cycle, but there are no equivalent yet with a globally identifying coloring.

Definition 8 (Identified Cycle). An identified cycle is a cycle C and a globally identifying coloration c of C .

In this paper, we will use a number of colors $L \geq 6$. Smaller values of L will be considered at the end of this article. We want to build a cycle as long as possible which admits a globally identifying coloring with at most L colors. In her Ph.D. dissertation, Parreau [11] noticed that universal cycles are identified cycles. Thus, Jackson's construction for universal cycles can be used to get identified cycles. In this paper we will focus on Jackson's construction in order to modify and improve it to get directly identified cycles.

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