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Packing degenerate graphs greedily

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Abstract

We prove that if $\mathcal{G}$ is a family of graphs with at most n vertices each, with constant degeneracy, with maximum degree at most $O(n/\log n)$, and with total number of edges at most $(1-o(1))\binom{n}{2}$, then $\mathcal{G}$ packs into the complete graph K_n.

Keywords: tree packing conjecture, graph packing, graph processes

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1 Introduction

A *packing* of a family $\mathcal{G} = \{G_1, \ldots, G_k\}$ of graphs into a host graph H is a colouring of the edges of H with the colours $0, 1, \ldots, k$ such that the edges of colour i form an isomorphic copy of G_i for each $1 \leq i \leq k$. Graph packing problems can be considered as a common generalisation of a number of important directions in Extremal Graph Theory. Here we focus on packings of large connected graphs that either exhaust all (*perfect packings*) or almost all the edges of the host graph H (*near-perfect packings*). The first and still the most famous problems are the *Tree Packing Conjectures*. In 1963 Ringel conjectured that if T is any $n+1$-vertex tree, then $2n+1$ copies of T pack into K_n, and in 1976 Gyárfás conjectured that if T_i is an i-vertex tree for each $1 \leq i \leq n$ then $\{T_1, \ldots, T_n\}$ packs into K_n. Since we have $(2n+1)e(T) = \binom{n}{2}$ and $\sum e(T_i) = \binom{n}{2}$, both conjectures ask for perfect packings. Despite many partial results both these problems were wide open until recently.

The first near-perfect packing result in this direction was obtained in [1], where it was shown that one can pack into K_n any family of trees whose maximum degree is at most Δ, whose order is at most $(1-\delta)n$, and whose total number of edges is at most $(1-\delta)\binom{n}{2}$, provided that n is sufficiently large given the constants $\Delta \in \mathbb{N}$ and $\delta > 0$. This approximately answers the Tree Packing Conjectures for bounded degree trees. Various generalisations were obtained in quick succession. The paper [6] shows that one can replace trees with graphs from any nontrivial minor-closed family. This was improved in [2] by allowing the graphs to be packed to be spanning. The paper [5] proves a near-perfect packing result for families of graphs with bounded maximum degree which are otherwise unrestricted. Both Tree Packing Conjectures for trees of bounded maximum degree were solved in [4]. The paper [3] gives near-perfect packing results for spanning trees, and for almost spanning trees, allowing the maximum degrees to be as big $O\big(n^{1/6}/\log^6 n\big)$, and $O\big(n/\log n\big)$, respectively. Our result is a near-perfect packing theorem for spanning graphs with bounded degeneracy and maximum degrees up to $O\big(n/\log n\big)$, extending the mentioned packing results. [5]

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[5] However, the main focus of [3] is on packing into random graphs, and [5] provides a packing result in the setting of the Regularity lemma.