

On quadrangulations and Stokes complexes

Amir-Hossein Bateni

LIX, École polytechnique, Palaiseau, France

Thibault Manneville

LIX, École polytechnique, Palaiseau, France

Vincent Pilaud

CNRS & LIX, École polytechnique, Palaiseau, France

Abstract

Stokes complexes consist of sets of mutually noncrossing diagonals of a convex polygon, that are in some sense compatible with a reference quadrangulation. Originally defined by Y. Baryshnikov (2001), they were recently revisited by F. Chapoton (2016) who proposed several conjectures. We settle two of these conjectures and study geometric realizations of Stokes complexes using compatibility vectors.

1 Introduction

Let $\mathcal{P}$ be a convex polygon with $2n + 4$ vertices labeled clockwise from 1 to $2n + 4$ with odd (resp. even) vertices colored black (resp. white). We only consider *bicolor diagonals* of $\mathcal{P}$ whose endpoints have different colors (oriented from their black endpoint to their white endpoint in all the paper), and *bicolor dissections* of $\mathcal{P}$, namely sets of mutually noncrossing bicolor diagonals. A *quadrangulation* is a maximal bicolor dissection, which thus separates $\mathcal{P}$

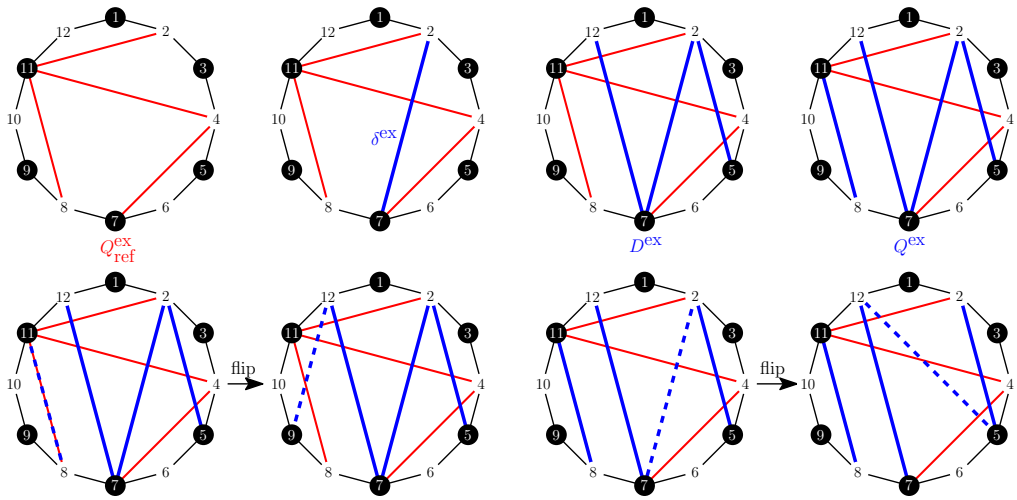


Figure 1. Top, left to right: a quadrangulation $Q_{\text{ref}}^{\text{ex}}$ of a polygon with 12 vertices, and a diagonal δ^{ex} , a dissection D^{ex} and a quadrangulation Q^{ex} that are all $Q_{\text{ref}}^{\text{ex}}$ -compatible. Bottom: a directed flip involving a reference diagonal (left) and a generic directed flip (right). Reference diagonals (when considered in $Q_{\text{ref}}^{\text{ex}}$) are red, other diagonals are blue. The diagonals exchanged in the flips are dashed.

into quadrangles. For a reference quadrangulation Q_{ref} , Y. Baryshnikov [1] defines the Q_{ref} -compatibility as follows. For a reference diagonal $\delta_{\text{ref}} \in Q_{\text{ref}}$, a diagonal δ of $\mathcal{P}$ is δ_{ref} -compatible if $\delta = \delta_{\text{ref}}$, or if δ and δ_{ref} do not cross, or if the pair of oriented diagonals $(\delta_{\text{ref}}, \delta)$ defines the positive orientation of the plane. The diagonal δ is Q_{ref} -compatible if it is compatible with all the diagonals of Q_{ref} , and a bicolor dissection is Q_{ref} -compatible if all its diagonals are. The Stokes complex of Q_{ref} is the simplicial complex $\mathcal{S}(Q_{\text{ref}})$ whose faces are Q_{ref} -compatible bicolor dissections. Y. Baryshnikov shows in [1] that this complex is a pseudomanifold of dimension $n - 1$, that is all its facets are quadrangulations of $\mathcal{P}$, and there is a notion of flip on Q_{ref} -compatible quadrangulations allowing to exchange a diagonal δ of such a quadrangulation Q for a unique other one δ' making $Q \triangle \{\delta, \delta'\}$ a Q_{ref} -compatible quadrangulation (see [3] for a precise description of this flip operation and Figure 1 for an illustration). In [3], F. Chapoton revisits Stokes complexes and introduces new related objects. He considers the Stokes poset $\mathcal{P}(Q_{\text{ref}})$, obtained as the transitive closure of the flip graph of Q_{ref} oriented as follows. The flip between two adjacent Q_{ref} -compatible quadrangulations Q and Q' with $Q \setminus \{\delta\} = Q' \setminus \{\delta'\}$ is oriented from Q to Q' if

- ◊ the diagonal δ is a reference diagonal (that is $\delta \in Q_{\text{ref}}$),
- ◊ or there is a reference diagonal $\delta_{\text{ref}} \in Q_{\text{ref}}$ satisfying

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