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Complementary cycles in regular bipartite tournaments: a proof of Manoussakis, Song and Zhang Conjecture

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Abstract

Let D be a k-regular bipartite tournament. We show that, for every even p with $4 \leq p \leq |V(D)| - 4$, D has a cycle C of size p such that $D \setminus C$ is Hamiltonian unless D is isomorphic to a special digraph, F_{4k}. This result proves a conjecture of Manoussakis, Song and Zhang.

Keywords: Cycle factor, Hamiltonian cycle, Regular bipartite tournament

1 Introduction

A cycle factor of a digraph D is a spanning subdigraph of D whose components are vertex-disjoint (directed) cycles. For some strictly positive integer k, a k-

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cycle factor of D is a cycle factor of D with k vertex-disjoint cycles; it can also be considered as a partition of D into k Hamiltonian digraphs. In particular, a 1-cycle factor is an Hamiltonian cycle of D. Finally, a $(n_1, \ldots, n_k)$-*cycle factor* is a k-cycle factor whose cycles are of size $n_1, \ldots, n_k$, where $n_1 + \ldots + n_k = |V(D)|$. When $k = 2$, two spanning disjoint cycles of a 2-cycle factor are called *complementary cycles*. Finding cycles of many lengths in different digraphs is a natural problem in Graph Theory [2]. For example, Moon proved in [8] that every vertex of a strong tournament is in a cycle of every length. More specifically about k-cycle factors in tournaments, Chen and *al.* proved in [4] that every k-connected tournament with at least $8k$ vertices contains a k-cycle factor. We can also mention the following result, due to Reid in [9] and Song in [10], which is that every 2-connected tournament with at least 6 vertices and not isomorphic to T_7, the Paley tournament, with 7 vertices with no transitive subtournament with 4 vertices, has a 2-cycle factor of lengths p and $|V(T)| - p$ for all p such that $3 \leq p \leq |V(T)| - 3$. Li and Shu finally refined the previous result by proving in [6] that any strong tournament with at least 6 vertices, a minimum out-degree or a minimum in-degree at least 3, and not isomorphic to T_7 has 2-cycle factor of length p and $|V(T)| - p$ for all p such that $3 \leq p \leq |V(T)| - 3$.

In this paper, we focus on cycle factors in k-regular bipartite tournaments, that is in orientations of complete bipartite graphs such that every vertex has an in-degree and an out-degree equal to k. Thus, notice that these digraphs have $4k$ vertices. The existing results concerning this class of digraphs try to extend what we know about cycle factors in tournaments. For example, Zhang and Song proved in [11] that any k-regular bipartite tournament with $k \geq 2$ has a 2-cycle factor. Moreover, Manoussakis, Song and Zhang conjectured in [12] the main Theorem of this article:

Theorem 1.1 *Let D be a k-regular bipartite tournament not isomorphic to F_{4k}. Then for every even p with $4 \leq p \leq |V(D)| - 4$, D has a 2-cycle factor of lengths p and $|V(D)| - p$.*

The digraph F_{4k} corresponds to the k-regular bipartite tournament consisting of four independent sets K, L, M and N each of cardinality k with all possible arcs from K to L, from L to M, from M to N and from N to K. In fact, every cycle of F_{4k} has length $0 \pmod 4$. For instance F_{4k} has no 2-cycle factor of length 6 and $|V(F_{4k})| - 6$. Zhang et *al.* proved their conjecture when $p = 4$ in their original paper [12]. In 2014 Bai, Li and He proved the conjecture for $p = 6$ in [1]. Notice that finding a 2-cycle factor with a cycle of prescribed length in a digraph guarantees us we can partition our digraph