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The Interactive Sum Choice Number of Graphs

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Abstract

We introduce a variant of the well-studied sum choice number of graphs, which we call the *interactive sum choice number*. In this variant, we request colours to be added to the vertices' colour-lists one at a time, and so we are able to make use of information about the colours assigned so far to determine our future choices. The interactive sum choice number cannot exceed the sum choice number and we conjecture that, except in the case of complete graphs, the interactive sum choice number is always strictly smaller than the sum choice number. In this paper we provide evidence in support of this conjecture, demonstrating that it holds for a number of graph classes, and indeed that in many cases the difference between the two quantities grows as a linear function of the number of vertices.

Keywords: List coloring, sum-choice number.

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1 Introduction

The *choice number* of a graph G is the minimum length of colour-list that must be assigned to each vertex of G so that, regardless of the choice of colours in these lists, there is certain to be a proper colouring of G in which every vertex is coloured with a colour from its list. A small subgraph of G which is, in some sense, "hard" to colour, can therefore force the choice number for G to be large, even if most of the graph is "easy" to colour. The *sum choice number* of G (written $\chi_{\mathrm{SC}}(G)$), introduced by Isaak [9], captures the "average difficulty" of colouring a graph: each vertex can now be assigned a different length of colour-list, and the aim is to minimise the sum of the list lengths (while still guaranteeing that there will be a proper list colouring for any choice of lists). A long odd cycle is an example of a graph where most of the graph is "easier" to colour than the choice number indicates.

For any graph $G = (V, E)$, we have $\chi_{\mathrm{SC}}(G) \leq |V| + |E|$: we can order the vertices arbitrarily and assign to each vertex $d^-(v) + 1$ colors, where $d^-(v)$ is the number of neighbors of v that are before it in the order, and colour greedily in this order. Graphs for which this so-called greedy bound is in fact equal to the sum choice number are said to be *sc-greedy*, and one of the main topics for research into the sum choice number has been the identification of families of graphs which are (or are not) sc-greedy; a lot of work has been done on the sum choice number of graphs (see, for example [2,6,7,8,10,11]), but relatively little is known.

In this paper we introduce a variation of the sum choice number, called the *interactive sum choice number* of G (written $\chi_{\mathrm{ISC}}(G)$), in which we do not have to determine in advance all of the lengths of the colour lists: at each step we ask for a new colour to be added to the colour list for some vertex of our choosing and, depending on what colours have been added to lists so far, we can adapt our strategy. It is clear that $\chi_{\mathrm{ISC}}(G) \leq \chi_{\mathrm{SC}}(G)$ for any graph G, as we can simply ask for the appropriate number of colours to be added to the list for each vertex without paying any attention to the colours that have been added so far. The natural question is then whether we are in fact able to improve on the sum choice number of G by exploiting partial information about the colour lists.

If $G = (V, E)$ is a complete graph, the answer to this question is no. To

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