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Embedding spanning bounded degree subgraphs in randomly perturbed graphs [5]

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Abstract

We study the model of randomly perturbed dense graphs, which is the union of any graph G_α with minimum degree αn and the binomial random graph $G(n,p)$. For $p = \omega(n^{-2/(\Delta+1)})$, we show that $G_\alpha \cup G(n,p)$ contains any single spanning graph with maximum degree Δ. As in previous results concerning this model, the bound for p we use is lower by a log-term in comparison to the bound known to be needed to find the same subgraph in $G(n,p)$ alone.

Keywords: random graphs, spanning subgraphs, thresholds

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1 Introduction and Result

1.1 Thresholds in $G(n,p)$

Let $G(n,p)$ be the binomial random graph model, where among n vertices each possible edge is chosen independently with probability p.

An important part of random graph theory is the understanding of threshold behaviour with respect to certain graph properties. We say that $\hat{p}$ is a threshold for a graph property $\mathcal{F}$ if $\mathbb{P}[G(n,p) \in \mathcal{F}] \to 0$ for $p = o(\hat{p})$ and $\mathbb{P}[G(n,p) \in \mathcal{F}] \to 1$ for $p = \omega(\hat{p})$. If the latter is true, then we say that $G(n,p)$ has the property $\mathcal{F}$ with high probability (whp) and that this $\hat{p}$ is an upper bound for the threshold. Containing a graph as a (not neccesarily induced) subgraph is a monotone property and therefore admits a threshold [7].

In the following we will focus on spanning subgraphs. In their early, seminal work Erdős and Rényi [10] determined the threshold for perfect matchings in $G(n,p)$, which is $\ln n/n$. Pósa [21] and Korŝunov [15] independently showed that the property of having a Hamilton cycle has the same threshold. Recently, there has been a lot of work on the threshold for a bounded degree spanning tree, where the current best bound, by the second author [18,19], is $p \geq \Delta \ln^5 n/n$. A breakthrough result was achieved by Johannson, Kahn and Vu [13] who showed that the (sharp) threshold for a $K_{\Delta+1}$-factor, that is $n/(\Delta+1)$ vertex-disjoint copies of $K_{\Delta+1}$, is given by

$$p_\Delta := (n^{-1} \ln^{1/\Delta} n)^{\frac{2}{\Delta+1}}.$$

Turning to a much more general class of graphs, let $\mathcal{F}(n,\Delta)$ be the family of graphs on n vertices with maximum degree at most Δ. For some absolute constant C, Alon and Füredi [3] proved that, if $p \geq C(\ln n/n)^{1/\Delta}$, then $G(n,p)$ contains a fixed graph from $\mathcal{F}(n,\Delta)$ whp. This is far from optimal and since the clique-factor is widely believed to be the hardest graph in $\mathcal{F}(n,\Delta)$ to embed, it is natural to state the following conjecture.

Conjecture 1.1 *If $\Delta > 0$, $F \in \mathcal{F}(n,\Delta)$ and $p = \omega(p_\Delta)$, then whp $G(n,p)$ contains a copy of F.*

For $\Delta = 2$, this conjecture was very recently solved by Ferber, Kronenberg and Luh [11], who in fact showed a stronger so-called universality statement, is finding all graphs of the class simultaneously. larger Δ, Riordan [22] gave a general result, which requires a probability larger by a factor of $n^{\Theta(1/\Delta^2)}$ from p_Δ. The current best result in this direction is the following almost spanning