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Almost partitioning 2-edge-colourings of 3-uniform hypergraphs with two monochromatic tight cycles

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Abstract

We show that any 2-colouring of the 3-uniform complete hypergraph $K_n^{(3)}$ on n vertices contains two disjoint monochromatic tight cycles of distinct colours covering all but $o(n)$ vertices of $K_n^{(3)}$. The same result holds if we replace tight cycles with loose cycles.

Keywords: Monochromatic cycle partitioning, tight cycles.

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1 Introduction

Given a complete r-edge-colouring of graph or hypergraph $\mathcal{K}$, the problem of partitioning the vertices of $\mathcal{K}$ into the smallest number of monochromatic cycles has received much attention. Central to this area has been an old conjecture of Lehel [2] stating that two monochromatic disjoint cycles in different colours are sufficient to partition the vertex set of the complete graph $\mathcal{K}_n$ on n vertices, for all n. This was confirmed for large n in [10] and [1], and more recently, for all n, by Bessy and Thomassé [3].

For $r \geq 3$, there exist r-edge-colourings of $\mathcal{K}_n$ which do not allow for a partition of the vertex set into r monochromatic cycles [11]. On the other hand, the currently best bound (see [6]) shows that $100r \log r$ monochromatic cycles are sufficient to partition the vertex set of $\mathcal{K}_n$.

The problem transforms in the obvious way to hypergraphs, considering r-edge-colourings of the k-uniform complete hypergraph $\mathcal{K}_n^{(k)}$ on n vertices and partitions into one of the many notions of cycles in hypergraphs. Here we deal with loose and tight cycles. Loose cycles are uniform hypergraphs with a cyclic ordering of its edges such that consecutive edges intersect in exactly one vertex and nonconsecutive edges have empty intersection. On the other hand, tight cycles are k-uniform hypergraphs with a cyclic ordering of its vertices such that the edges are all the sets of k consecutive vertices. For loose cycles, the best bound due to Sárközy in [12] shows that every r-edge-colouring of $\mathcal{K}_n^{(k)}$ admits a partition of its vertices into at most $50rk\log(rk)$ monochromatic loose cycles. Concerning tight cycles, to our best knowledge, nothing is known. We refer the reader to [5] for related results.

Our main result establishes an approximate version of the problem for the case of 3-uniform hypergraphs and two colours.

Theorem 1.1 *For every $\eta > 0$ there exists n_0 such that if $n \geq n_0$ then every 2-coloring of the edges of the complete 3-uniform hypergraph $\mathcal{K}_n^{(3)}$ admits two vertex-disjoint monochromatic tight cycles, of distinct colours, which cover all but at most ηn vertices.*

We note that a 3-uniform tight cycle on n vertices contains a loose cycle if n is even. The proof of Theorem 1.1 guarantees that the two tight cy-

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