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Computing the number of realizations of a Laman graph

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Abstract

Laman graphs model planar frameworks which are rigid for a general choice of distances between the vertices. There are finitely many ways, up to isometries, to realize a Laman graph in the plane. In a recent paper we provide a recursion formula for this number of realizations using ideas from algebraic and tropical geometry. Here, we present a concise summary of this result focusing on the main ideas and the combinatorial point of view.

Keywords: Laman graph, minimally rigid graph, tropical geometry, euclidean embedding, graph realization

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1 Introduction

Suppose that we are given a graph G with edges E. We consider the set of all possible realizations of the graph in the plane such that the lengths of the edges coincide with some edge labeling $\lambda\colon E \to \mathbb{R}_{\geq 0}$. Edges and vertices are allowed to overlap in such a realization. For example, suppose that G has four vertices and is a complete graph minus one edge. Figure 1 shows all possible realizations of G up to rotations and translations, for a given edge labeling. We say that a property holds for a general edge labeling if it holds for all

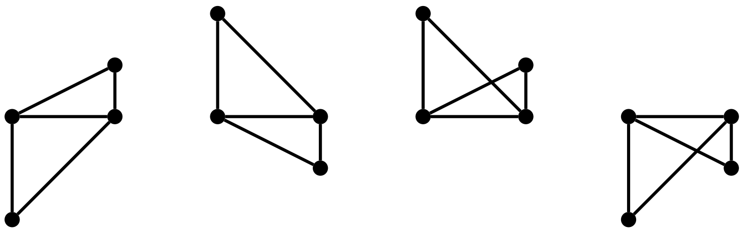

Figure 1. Realizations of a graph up to rotations and translations.

edge labelings belonging to a dense open subset of the vector space of all edge labelings. Here we address the following problem:

For a given graph determine the number of realizations up to rotations and translations for a general edge labeling.

The realizations of a graph can be considered as structures in the plane, which are constituted by rods connected by rotational joints. If a graph with an edge labeling admits infinitely (finitely) many realizations up to rotations and translations, then the corresponding planar structure is flexible (rigid).

The study of rigid structures, also called frameworks, was originally motivated by mechanics and architecture, and it goes back at least to the 19th century. Nowadays, there is still a considerable interest in rigidity theory [4] due to various applications in natural science and engineering.

A graph is called generically rigid (or isostatic) if a general edge labeling yields a rigid realization. No edge in a generically rigid graph can be removed without losing rigidity, that is why such graphs are also called minimally rigid in the literature. The complete graph on four vertices K_4 is for instance not considered to be generically rigid, since for a general choice of edge lengths it will not have a realization. Gerard Laman [6] characterized the property of generic rigidity in terms of the number of edges and vertices of the graph and its subgraphs, hence such objects are also known as *Laman graphs.*

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