Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 61 (2017) 223–229 www.elsevier.com/locate/endm

Combinatorial Problems on H-graphs [⋆]

Steven Chaplick [1]

Lehrstuhl für Informatik I, Universität Würzburg, Germany

Peter Zeman [2,3]

Department of Applied Mathematics, Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic

Abstract

Biró, Hujter, and Tuza introduced the concept of H-graphs (1992), intersection graphs of connected subgraphs of a subdivision of a fixed graph H. They naturally generalize many important classes of graphs. We continue their study by considering coloring, clique, and isomorphism problems. We show that if H contains a certain multigraph as a minor, then H-graphs are GI-complete and the clique problem is APX-hard. Also, when H is a cactus the clique problem can be solved in polynomial time and when a graph G has a Helly H-representation, the clique problem can be solved in polynomial time. We use treewidth to show that both the k-clique and list k-coloring problems are FPT on H-graphs. These results also apply to *treewidth-bounded* classes where treewidth is bounded by a function of the clique number.

Keywords: intersection graphs, clique, isomorphism, coloring, treewidth.

[⋆] The accepted version of this paper before arbitrary shortening due a **post-acceptance** restriction to 7 pages ENDM is at `https://arxiv.org/abs/1706.00575`.

[1] Email: `first.last@uni-wuerzburg.de`

[2] Email: `zeman@kam.mff.cuni.cz`

[3] The author was supported by GAUK 1334217.

http://dx.doi.org/10.1016/j.endm.2017.06.042

1 Introduction

An intersection representation of a graph assigns a set to each vertex and uses intersections of those sets to encode its edges. More formally, an intersection representation $\mathcal{R}$ of a graph G is a collection of sets $\{R_v\}_{v \in V(G)}$ such that $R_u \cap R_v \neq \emptyset$ if and only if $uv \in E(G)$. Many important classes of graphs arise from restricting the sets R_v to geometric objects (e.g., intervals, convex sets).

We study *H-graphs*, intersection graphs of connected subsets of a fixed topological pattern given by a graph H, introduced by Biró, Hujter, and Tuza [1]. We obtain new algorithmic results on clique, coloring, and isomorphism problem. In a companion paper [5], we studied recognition and dominating set problems on H-graphs. We begin with related graph classes. Further details on these clases can be found in [3].

Interval graphs (INT) form one of the most studied and well-understood classes of intersection graphs. In an *interval representation*, each set R_v is a closed interval of the real line; see Fig. 1a. A graph is *chordal* when it does not have an induced cycle of length at least four. Equivalently, a graph is chordal if and only if it can be represented as an intersection graph of subtrees of some tree; see Fig. 1b. This implies that INT is a subclass of the chordal graphs (CHOR). *Circular-arc graphs* (CARC) generalize interval graphs by having each set R_v be an arc of a circle. A graph G is a *Helly circular-arc graph* if the collection of circular arcs $\mathcal{R} = \{R_v\}_{v \in V(G)}$ satisfies *Helly property*, i.e., in each sub-collection of pairwise intersecting arcs, the arcs have a common point.

H-graphs. Biró, Hujter, and Tuza [1] introduced *H-graphs*. Let H be a fixed graph. A graph G is an *intersection graph of H* if it is an intersection graph of connected subgraphs of H, i.e., the assigned subgraphs H_v and H_u of H share a vertex if and only if $uv \in E(G)$.

A *subdivision* H' of a graph H is obtained when the edges of H are replaced by internally disjoint paths of arbitrary lengths. A graph G is a *topological intersection graph* of H if G is an intersection graph of a subdivision H' of H. We say that G is an *H-graph* and the collection $\{H'_v : v \in V(G)\}$ of connected subgraphs of H' is an *H-representation* of G. The class of all

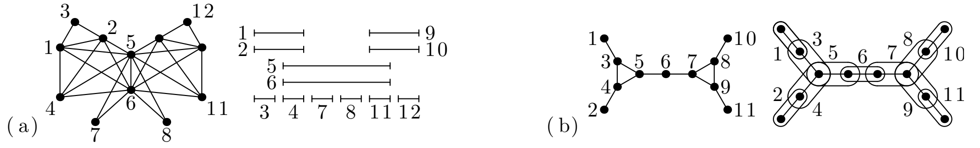

Fig. 1. (a) An interval graph and one of its interval representation. (b) A chordal graph and one of its representation as an intersection graph of subtrees of a tree.