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Bijections for walks ending on an axis, using open arc diagrams

Julien Courtiel[1], Éric Fusy[2], Mathias Lepoutre[2], Marni Misna[3]

[1] *LIPN, Université Paris 13, Villetaneuse, France*
[2] *LIX, École polytechnique, Palaiseau, France*
[3] *Department of Mathematics, Simon Fraser University, Burnaby, Canada*

Abstract

In the study of lattice walks there are several examples of enumerative equivalences which amount to a trade-off between domain and endpoint constraints. We present a family of such bijections for simple walks in Weyl chambers which use arc diagrams in a natural way. One consequence is a set of new bijections for standard Young tableaux of bounded height. A modification of the argument in two dimensions yields a bijection between Baxter permutations and walks ending on an axis, answering a recent question of Burrill et al. (2016).

Keywords: lattice paths, excursions, Schnyder woods, Dyck paths, Weyl chambers, Young tableaux, open arc diagrams.

1 Introduction

In the context of directed 2D lattice paths with unit steps, there is a classic bijection between *meanders* and *bridges* of equal length. This maps lattice

[1] Email: `courtiel@lipn.univ-paris13.fr`
[2] Email: `{fusy,mathias.lepoutre}@lix.polytechnique.fr`
[3] Email: `mmishna@sfu.ca`

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walks with steps $(1, 1)$ and $(1, -1)$ starting at the origin, staying above the x-axis (meanders) to those ending at height zero (bridges) – see Figure 1. This example illustrates a common trade-off in lattice walks between domain

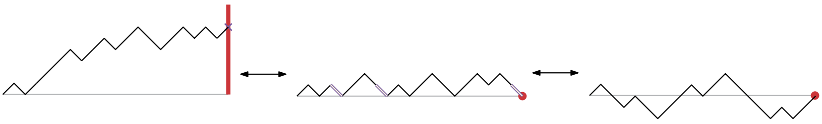

Fig. 1. An example of the classical bijection between meanders and bridges, via (decorated) Dyck walks.

constraints and endpoint constraints [8,3]. Note that the natural bijection illustrated in Figure 1 proceeds via an intermediate class of walks (Dyck paths) where both the stronger domain and endpoint restrictions are imposed, and the elements of this class carry additional "decorations" (here, marked down-steps reaching the x-axis).

In this work, we use a similar strategy to describe bijections for walks in higher dimensions. For the two classical step sets known widely as simple walks and hesitating walks – defined at the beginnings of Sections 3 and 4 respectively – we find explicit bijections that exchange a domain constraint with an endpoint constraint. For example, in two dimensions we map simple walks in the quadrant $\{(x, y) : x \geq 0,\ y \geq 0\}$ ending at the origin (*excursions*[4]) to walks in the octant $\{(x, y) : x \geq y \geq 0\}$ ending on the x-axis (*axis-walks*).

For both step sets, these bijections pass through decorated excursions restricted to the octant. Deciding exactly how to mark the steps in the decorated intermediary is less obvious than the Dyck path example. We do this by using *open arc diagrams* that are associated to the walks via the robust bijection of Chen *et al.* [5] and its extension to open arc diagrams due to Burrill *et al.* [4]. In their full generality, these bijections map open diagrams with no $(k + 1)$-crossing[5] to walks in the k-dimensional Weyl chamber of type C, $\{(x_1, \dots, x_k) : x_1 \geq \cdots \geq x_k \geq 0\}$, that end on the x_1-axis, where the number of open arcs gives the abscissa of the endpoint. It is at the level of arc diagrams that the marking of the object is easiest to describe: we map walks that end on the x-axis to open arc diagrams, mark the location of the open arcs, remove them, and then apply the inverse bijection to get marked excursions. The schematic outline of our core idea is illustrated in Figure 2.

[4] More generally, we use the term excursion for the set of walks with a prescribed start and end point. When they are not specified, the prescribed start and end are assumed to be the origin.

[5] A k-crossing is a set of k mutually crossing arcs.

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