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On the List Coloring Version of Reed's Conjecture

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Abstract

The chromatic number of a graph is bounded below by its clique number and from above by its maximum degree plus one. In 1998, Reed conjectured that the chromatic number is at most halfway in between these trivial lower and upper bounds. Moreover, Reed proved that its at most some non-trivial convex combination of the two bounds. In 2012, King and Reed produced a short proof that, provided the maximum degree is large enough, a combination of 1/130,000 suffices. Recently Bonamy, Perrett, and the second author improved this to 1/26.

It is natural to wonder if similar results hold for the list chromatic number. Unfortunately, previous techniques for ordinary coloring do not extend to list coloring. In this paper, we overcome these hurdles by introducing several new ideas. Our main result is that the list chromatic number is at most some non-trivial convex combination of the clique number and the maximum degree plus one. Furthermore,

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we show that for large enough maximum degree, that a combination of 1/13 suffices. Note that this also improves on the best known results for ordinary coloring.

Keywords: Graph Coloring, List Coloring, Reed's Conjecture

1 Introduction

While one of the oldest topics in graph theory, graph coloring is still an active area of interest for many researchers with a host of unsolved fundamental problems. Let us recall that a *k-coloring* of a graph G is an assignment of colors 1 to k to vertices of a graph G such that no two adjacent vertices receive the same color. The *chromatic number* $\chi(G)$ is the minimum natural number k such that G has a k-coloring. One of these fundamental questions centers on the best known upper and lower bounds on the chromatic number of sparse graphs. A straightforward greedy coloring procedure provides a trivial upper bound for $\chi(G)$ of $\Delta(G) + 1$, where $\Delta(G)$ denotes the maximum degree of G. Similarly, a trivial lower bound on $\chi(G)$ is the clique number $\omega(G)$, which is the largest number of pairwise adjacent vertices in G. A fundamental yet natural question then is whether the chromatic number is closer to one bound than the other.

In 1998, Reed [6] conjectured that, up to rounding, the chromatic number of a graph is at most the average of its trivial upper and lower bounds as follows.

Conjecture 1.1 [6] *If G is a graph, then* $\chi(G) \leq \lceil \frac{1}{2}(\Delta(G) + 1 + \omega(G)) \rceil$.

In support of his conjecture, Reed [6] proved that the chromatic number can be bounded above by some non-trivial convex combination of ω and $\Delta + 1$ as follows.

Theorem 1.2 [6] *There exists $\varepsilon > 0$ such that for every graph G, we have*

$$\chi(G) \leq \lceil (1 - \varepsilon)\,(\Delta(G) + 1) + \varepsilon\omega(G) \rceil.$$

King and Reed [4] subsequently gave a shorter proof of Theorem 1.2. Furthermore, they showed that, for large Δ, $\varepsilon \leq \frac{1}{320e^6}$ suffices. More recently,

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