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Two-layer Drawings of Bipartite Graphs

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Abstract

We give a polynomial-time algorithm to decide whether a connected bipartite graph admits a two-layer drawing in the plane such that a specified subset of pairs of disjoint edges cross. We consider the problem of deciding whether there exists such a drawing in which a specified subset of triples of pairwise crossing edges are concurrent. We give a necessary condition for the same and conjecture that it is sufficient.

Keywords: abstract topological graph, two-layer drawing, bipartite graph

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1 Introduction

An *abstract topological graph* is a graph $G(V, E)$ together with a collection F of unordered pairs of disjoint edges. A *realization* of an abstract topological graph is a drawing of the graph G in the plane satisfying the following properties.

(i) Vertices are represented by distinct points in the plane.

(ii) An edge is represented by the line segment joining the points representing its endvertices and it does not contain any point representing a vertex in its interior.

(iii) The line segments representing disjoint edges e_1, e_2 intersect in their interior if and only if $\{e_1, e_2\} \in F$. In this case, we say the edges cross in the drawing.

A *weak realization* of an abstract topological graph is a drawing in which pairs of disjoint edges in F cross, but pairs of edges not in F may or may not cross.

In general, deciding whether an abstract topological graph is realizable, or weakly realizable, is NP-hard [6]. We consider only bipartite graphs, with a fixed bipartition of the vertices, and a special kind of drawing called a *2-layer drawing*. In such a drawing, all vertices in one part are represented by distinct points contained in a horizontal line, with distinct lines used for the two parts. Such drawings have been extensively studied [2]. A bipartite abstract topological graph G has a 2-layer realization if there exists a 2-layer drawing of G that realizes it. Note that if the graph G is a matching, deciding whether it has a 2-layer realization is equivalent to the problem of recognizing permutation graphs [4].

We give a polynomial-time algorithm to decide whether a given abstract topological bipartite graph has a 2-layer realization. For simplicity, in this abstract, we consider only the case of connected bipartite graphs, and leave the details of the general case for the full version.

In contrast, the problem of deciding whether a connected bipartite abstract topological graph has a weak 2-layer realization is known to be NP-complete [3].

Suppose we are given a 2-layer drawing of a bipartite graph G. Let F' be a collection of 3 element subsets of pairwise crossing edges. We say the collection F' is *realizable* if there exists a 2-layer drawing of G, with the same ordering of the vertices as in the given drawing, such that the line segments representing pairwise crossing edges e_1, e_2, e_3 are concurrent, if and only if $\{e_1, e_2, e_3\} \in F'$.