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First steps in combinatorial optimization on graphons: Matchings

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Abstract

Much of discrete optimization concerns problems whose underlying structures are graphs. Here, we translate the theory around the maximum matching problem to

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the setting of graphons. We study continuity properties of the thus defined matching ratio, limit versions of matching polytopes and vertex cover polytopes, and deduce a version of the LP duality for the problem of maximum fractional matching in the graphon setting.

Keywords: graphon, graph limits, matching, combinatorial optimization

1 Introduction

The study of matchings is central both in graph theory and in theoretical computer science. It has three sides: structural, polyhedral, and algorithmic. The structural part of the theory includes results such as the Gallai–Edmonds matching theorem. The study of polyhedral aspects — which include the geometry of the matching polytope, the vertex cover polytope — is much motivated by linear programming. Finally, algorithmic questions include, e.g., the study of fast algorithms for finding the maximum matching, or are motivated by theory related to property testing and parameter estimation. These three sides are very much intertwined.

We initiate and study concepts related to matchings in the setting of graphons. Graphons are analytic object which capture properties of large graphs. They were introduced in [1,6] as limit representation of large dense graphs. Since then they have played a key role in extremal graph theory, theory of random graphs, and other parts of mathematics. While this announcement is rather dry due to space constraints, we believe that it will

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The papers containing the full description of the results and detailed proofs can be found in [4,2].

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