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3-uniform hypergraphs and linear cycles

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Abstract

We continue the work of Gyárfás, Győri and Simonovits [4], who proved that if a 3-uniform hypergraph H with n vertices has no linear cycles, then its independence number $\alpha \geq \frac{2n}{5}$. The hypergraph consisting of vertex disjoint copies of complete hypergraphs K_5^3 shows that equality can hold. They asked whether α can be improved if we exclude K_5^3 as a subhypergraph and whether such a hypergraph is 2-colorable.

We answer these questions affirmatively. Namely, we prove that if a 3-uniform linear-cycle-free hypergraph H, doesn't contain K_5^3 as a subhypergraph, then it is 2-colorable. This result clearly implies that $\alpha \geq \lceil \frac{n}{2} \rceil$. We show that this bound is sharp.

Gyárfás, Győri and Simonovits also proved that a linear-cycle-free 3-uniform hypergraph contains a vertex of strong degree at most 2. In this context, we show

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that a linear-cycle-free 3-uniform hypergraph has a vertex of degree at most $n-2$ when $n \geq 10$.

Keywords: linear cycle, loose cycle, independence number

1 Introduction

A hypergraph $H = (V, E)$ is k colorable if there is a coloring of the vertices of H with k colors such that there is no monochromatic hyperedge in H. Throughout the article, we mostly use the terminology introduced in [4].

Definition 1.1 A *linear tree* is a hypergraph obtained from a vertex by repeatedly adding hyperedges that intersect the previous hypergraph in exactly one vertex. A *linear path* is a linear tree built so that the next hyperedge always intersects the previous hyperedge in a vertex of degree one.

A *linear cycle* is obtained from a linear path of at least two edges, by adding an edge that intersects the first and the last edges of the linear path in one of their degree one vertices.

A *skeleton* T in H is a linear subtree of H which cannot be extended to a larger linear subtree by adding a hyperedge e of H for which $|e \cap V(T)| = 1$.

An independent set in H is a set of vertices containing no hyperedge of H. More precisely, if I is an independent set of H, then there is no $e \in E(H)$ such that $e \subseteq I$. Let $\alpha(H)$ denote the size of the largest independent set in H. Gyárfás, Győri and Simonovits [4] initiated the study of linear-cycle-free hypergraphs by showing:

Theorem 1.2 (Gyárfás, Győri, Simonovits [4]) *If H is a 3-uniform hypergraph on n vertices without linear cycles, then it is 3-colorable. Moreover, $\alpha(H) \geq \frac{2n}{5}$.*

If the hypergraph does not contain the complete 3-uniform hypergraph K_5^3 as a subhypergraph then a stronger theorem can be proved, answering a question of Gyárfás, Győri and Simonovits.

Theorem 1.3 (Ergemlidze, Győri, Methuku [2]) *If a 3-uniform linear-cycle-free hypergraph H doesn't contain K_5^3 as a subhypergraph, then it is 2-colorable.*

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