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Gaps in full homomorphism order

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Abstract

We characterise gaps in the full homomorphism order of finite graphs.

Keywords: graph homomorphism, full homorphism, homomorphism order, gap, full homomorphism duality, point determining graph

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1 Introduction

For given graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$ a *homomorphism* $f : G \to H$ is a mapping $f : V_G \to V_H$ such that $\{u, v\} \in E_G$ implies $\{f(u), f(v)\} \in E_H$. (Thus it is an edge preserving mapping.) The existence of a homomorphism $f : G \to H$ is traditionally denoted by $G \to H$. This allows us to consider the existence of a homomorphism, $\to$, to be a (binary) relation on the class of graphs. A homomorphism f is *full* if $\{u, v\} \notin E_G$ implies $\{f(u), f(v)\} \notin E_H$. (Thus it is an edge and non-edge preserving mapping). Similarly we will denote by $G \xrightarrow{F} H$ the existence of a full homomorphism $f : G \to H$.

As it is well known, the relations $\to$ and $\xrightarrow{F}$ are reflexive and transitive. Thus the existence of a homomorphism as well as the existence of full homomorphisms induces a quasi-order on the class of all finite graphs. We denote the quasi-order induced by the existence of homomorphisms and the existence of full homomorphism on finite graphs by $(\mathsf{Graphs}, \leq)$ and $(\mathsf{Graphs}, \leq^F)$ respectively. (Thus when speaking of orders, we use $G \leq H$ in the same sense as $G \to H$ and $G \leq^F H$ in the sense $G \xrightarrow{F} H$.)

These quasi-orders can be transformed into partial orders by choosing a particular representative for each equivalence class. In the case of graph homomorphism such representative is up to isomorphism unique vertex minimal element of each class, the *(graph) core*. In the case of full homomorphisms we will speak of *F-core*.

The study of homomorphism order is a well established discipline and one of main topics of nowadays classical monograph of Hell and Nešetřil [5]. The order $(\mathsf{Graphs}, \leq^F)$ is a topic of several publications [9,2,4,1,3] which are primarily concerned about the full homomorphism equivalent of the homomorphism duality [7].

In this work we further contribute to this line of research by characterising *F-gaps* in $(\mathsf{Graphs}, \leq^F)$. That is pairs of non-isomorphic F-cores $G \leq^F H$ such that every F-core H', $G \leq^F H' \leq^F H$, is isomorphic either to G or H.

Theorem 1.1 *If G and H are F-cores and (G, H) is an F-gap, then G can*

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