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Families with no matchings of size s

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Abstract

Let $k \geqslant 2$, $s \geqslant 2$ be positive integers. Let $[n]$ be an n-element set, $n \geqslant ks$. Subsets of $2^{[n]}$ are called *families*. If $\mathcal{F} \subset \binom{[n]}{k}$, then it is called *$k$-uniform*. What is the maximum size $e_k(n,s)$ of a k-uniform family without s pairwise disjoint members? The well-known Erdős Matching Conjecture would provide the answer for all n, k, s in the above range. For $n > 2ks$ it is known that the maximum is attained by $\mathcal{A}_1(T) := \{A \subset [n] : |A| = k, A \cap T \neq \emptyset\}$ for some fixed $(s-1)$-element set $T \subset X$. We discuss recent progress on this problem. In particular, our recent stability result states that for $n > (2 + o(1))ks$ and a k-uniform family $\mathcal{F}$, $\mathcal{F} \not\subseteq \mathcal{A}_1(T)$, then $|\mathcal{F}|$ is considerably smaller.

This result is applied to obtain the corresponding anti-Ramsey numbers in a wide range.

Removing the condition of uniformness, we arrive at another classical problem of Erdős, which was solved by Kleitman for $n \equiv 0$ or $-1 \pmod s$. We succeeded

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in resolving this long-standing problem for $n \equiv -2 \pmod{s}$ via a new averaging technique which might prove useful in various other situations.

Keywords: set families, matchings, Erdős Matching Conjecture, forbidden subconfigurations

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1 Introduction

Put $[n] := \{1, 2, \ldots, n\}$ and let $2^{[n]}$ denote the power set of $[n]$. A subset $\mathcal{F} \subset 2^{[n]}$ is called a *family of subsets of* $[n]$, or simply a *family*. For $0 \leqslant k \leqslant n$ we use the notation $\binom{[n]}{k} := \{H \subset [n] : |H| = k\}$.

The maximum number of pairwise disjoint members of a family $\mathcal{F}$ is denoted by $\nu(\mathcal{F})$ and is called the *matching number* of $\mathcal{F}$. Let us define the following two quantities for $n, k, s \geqslant 2$:

$$e(n,s) := \max\big\{|\mathcal{F}| : \mathcal{F} \subset 2^{[n]}, \nu(\mathcal{F}) < s\big\},$$
$$e_k(n,s) := \max\Big\{|\mathcal{F}| : \mathcal{F} \subset \binom{[n]}{k}, \nu(\mathcal{F}) < s\Big\}.$$

For $s = 2$ both quantities were determined by Erdős, Ko and Rado [4]: $e(n,2) = 2^{n-1}$, $e_k(n,2) = \binom{n-1}{k-1}$ for $n \geqslant 2k$. For $s \geqslant 3$ this becomes a much harder task. Let us first discuss the k-uniform problem.

2 The uniform case. Stability

The following families are the natural candidates for being an extremal family of k-sets with no $(s+1)$-matching:

$$\mathcal{A}_i^{(k)}(n,s) := \Big\{A \in \binom{[n]}{k} : \big|A \cap [(s+1)i - 1]\big| \geqslant i\Big\}, \quad 1 \leqslant i \leqslant k. \tag{1}$$

Conjecture 1 (Erdős Matching Conjecture [2]) *For $n \geqslant (s+1)k$ we have*

$$e_k(n, s+1) = \max\big\{|\mathcal{A}_1^{(k)}(n,s)|, |\mathcal{A}_k^{(k)}(n,s)|\big\}. \tag{2}$$

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