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Covering and tiling hypergraphs with tight cycles

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Abstract

Given $3 \leq k \leq s$, we say that a k-uniform hypergraph C_s^k is a tight cycle on s vertices if there is a cyclic ordering of the vertices of C_s^k such that every k consecutive vertices under this ordering form an edge. We prove that if $k \geq 3$ and $s \geq 2k^2$, then every k-uniform hypergraph on n vertices with minimum codegree at least $(1/2 + o(1))n$ has the property that every vertex is covered by a copy of C_s^k. Our result is asymptotically best possible for infinitely many pairs of s and k, e.g. when s and k are coprime.

A perfect C_s^k-tiling is a spanning collection of vertex-disjoint copies of C_s^k. When s is divisible by k, the problem of determining the minimum codegree that guarantees a perfect C_s^k-tiling was solved by a result of Mycroft. We prove that if $k \geq 3$ and $s \geq 5k^2$ is not divisible by k and s divides n, then every k-uniform hypergraph on n vertices with minimum codegree at least $(1/2 + 1/(2s) + o(1))n$ has a perfect C_s^k-tiling. Again our result is asymptotically best possible for infinitely many pairs of s and k, e.g. when s and k are coprime with k even.

Keywords: hypergraphs, tiling, covering, tight cycles, codegree threshold

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1 Introduction

A *hypergraph* $\mathcal{H} = (V(\mathcal{H}), E(\mathcal{H}))$ consists of a vertex set $V(\mathcal{H})$ and an edge set $E(\mathcal{H})$, where each edge $e \in E(\mathcal{H})$ is a subset of $V(\mathcal{H})$. Given a set V and a positive integer k, $\binom{V}{k}$ denotes the set of subsets of V with size exactly k. We say that a hypergraph $\mathcal{H}$ is *k-uniform* if $E(\mathcal{H}) \subseteq \binom{V(\mathcal{H})}{k}$, and we abbreviate '$k$-uniform hypergraphs' to *k-graphs*. Note that 2-graphs are usually known simply as *graphs*.

Given a hypergraph $\mathcal{H} = (V(\mathcal{H}), E(\mathcal{H}))$ and a set $S \subseteq V(\mathcal{H})$, let $\deg_{\mathcal{H}}(S)$ denote the number of edges of $\mathcal{H}$ containing the set S. We denote by $\delta_s(\mathcal{H})$ the *minimum s-degree of* $\mathcal{H}$, that is, the minimum of $\deg_{\mathcal{H}}(S)$ over all s-element sets $S \in \binom{V(\mathcal{H})}{s}$. Note that $\delta_0(\mathcal{H})$ is equal to the number of edges of $\mathcal{H}$. Also, $\delta_{k-1}(\mathcal{H})$ and $\delta_1(\mathcal{H})$ are referred to as the *minimum codegree* and the *minimum vertex degree* of $\mathcal{H}$, respectively.

A typical question in extremal graph theory is: what is the smallest δ such that every k-graph $\mathcal{H}$ on n vertices with $\delta_s(\mathcal{H}) \geq \delta$ has property P? In this work we focus our study on two different problems: the tiling problem and the covering problem.

1.1 Tiling thresholds

Let $\mathcal{H}$ and F be k-graphs. An *F-tiling* in $\mathcal{H}$ is a set of vertex-disjoint copies of F. An F-tiling is *perfect* if it spans the vertex set of $\mathcal{H}$. Note that a perfect F-tiling is also known as *F-factor* and *perfect F-matching*. For all $i, n \in \mathbb{Z}$ with $0 \leq i < k$, define the *ith-degree tiling threshold* $t_i(n, F)$ to be the maximum of $\delta_i(\mathcal{H})$ over k-graphs $\mathcal{H}$ on n vertices without a perfect F-tiling. Note that if $n \not\equiv 0 \bmod |V(F)|$ then a perfect F-tiling cannot exist and so, $t_i(n, F) = \binom{n-i}{k-i}$. Hence we will always assume that $n \equiv 0 \bmod |V(F)|$ whenever we discuss $t_i(n, F)$.

In the graph case, the tiling thresholds are well understood. Hajnal and Szemerédi [5] showed that $t_1(n, K_t) = (1 - 1/t)n - 1$ for all $t \geq 2$. For a general graph F, Kühn and Osthus [8] determined $t_1(n, F)$ up to an additive constant depending only on F.

For $k \geq 3$, Kühn and Osthus [8] determined $t_{k-1}(n, K_k^k)$ asymptotically and

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