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Characteristic polynomials of production matrices for geometric graphs

Clemens Huemer [1]

Departament de Matemàtiques
Universitat Politècnica de Catalunya
Barcelona, Spain

Alexander Pilz [2]

Department of Computer Science
ETH Zürich
Switzerland

Carlos Seara [1]

Departament de Matemàtiques
Universitat Politècnica de Catalunya
Barcelona, Spain

Rodrigo I. Silveira [1]

Departament de Matemàtiques
Universitat Politècnica de Catalunya
Barcelona, Spain

Abstract

An $n \times n$ production matrix for a class of geometric graphs has the property that the numbers of these geometric graphs on up to n vertices can be read off from

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the powers of the matrix. Recently, we obtained such production matrices for non-crossing geometric graphs on point sets in convex position [6]. In this note, we determine the characteristic polynomials of these matrices. Then, the Cayley-Hamilton theorem implies relations among the numbers of geometric graphs with different numbers of vertices. Further, relations between characteristic polynomials of production matrices for geometric graphs and Fibonacci numbers are revealed.

Keywords: geometric graph, production matrix, Fibonacci number, Riordan array

1 Introduction

A *geometric graph* on a point set $\mathcal{S}$ in the plane is a graph with vertex set $\mathcal{S}$ whose edges are straight-line segments with endpoints in $\mathcal{S}$. It is called *non-crossing* if no two edges intersect except at common endpoints. Here, we consider non-crossing geometric graphs on sets $\mathcal{S}$ of n points in convex position for the following graph classes: triangulations, matchings, spanning trees, forests, spanning paths, and all geometric graphs on n vertices. The numbers of these graphs are well known, see for instance the work of Flajolet and Noy [4]. Recently, in [6], we counted such geometric graphs by using an $n \times n$ matrix A_n, called *production matrix*, associated to the graph class. The numbers of these graphs on a certain number of vertices are then given by (a column of) powers of A_n. In order to derive a production matrix, first the graphs on $i \leq n$ vertices are partitioned according to the degree of a specified root vertex. Each part is counted in the elements of an n-element integer vector $\vec{v}^i$, and hence the sum of the elements gives the number of geometric graphs on i vertices. The *production matrix* A_n is such that $\vec{v}^{i+1} = A_n \vec{v}^i = A_n^{i+1-c} \vec{v}^c$, when starting with a vector $\vec{v}^c$ for a constant number of vertices, which will usually be $(1, 0, \ldots, 0)^\top$. To find the matrix A_n, the graphs are implicitly arranged in a tree structure (called *generating tree*), s.t., for each graph on i vertices and with root degree j, the number of its descendants on $i+1$ vertices with root degree ℓ (for each ℓ) is known. Generating trees are the basis of the *ECO method* [1], and have been used to obtain matrix representations for combinatorial objects [3,8]. Here we omit how the production matrices for geometric graphs are obtained, and only refer to works by Hurtado and Noy [7] for a generating tree of triangulations, and by Hernando et al. [5] for a generating tree of spanning trees; and also to [6]. Figure 1 shows the obtained

[1] Emails: {clemens.huemer,carlos.seara,rodrigo.silveira}@upc.edu
[2] Email: alexander.pilz@inf.ethz.ch