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A skew version of the Loebl–Komlós–Sós conjecture

Tereza Klimošová [1,2]

Department of Applied Mathematics, Faculty of Mathematics and Physics
Charles University
Prague, Czech Republic

Diana Piguet [1,3]

Institute of Computer Science
The Czech Academy of Sciences
Prague, Czech Republic

Václav Rozhoň [1,4]

Faculty of Mathematics and Physics
Charles University
and
Institute of Computer Science
The Czech Academy of Sciences
Prague, Czech Republic

Abstract

Loebl, Komlós, and Sós conjectured that any graph such that at least half of its vertices have degree at least k contains every tree of order at most $k+1$. We propose a skew version of this conjecture. We consider the class of trees of order at most $k+1$ of given *skew*, that is, such that the sizes of the colour classes of the trees have a given ratio. We show that our conjecture is asymptotically correct for

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dense graphs. The proof relies on the regularity method. Our result implies bounds on Ramsey number of several trees of given skew.

Keywords: extremal graph theory, trees, Loebl–Komlós–Sós conjecture, regularity lemma

1 Introduction and results

Many problems in extremal graph theory ask whether a certain density condition imposed on a host graph forces the containment of a given subgraph H. Typically, the density condition is expressed by the average or minimum degree. For example, the Erdős-Stone Theorem [3] essentially determines the average degree condition guaranteeing the containment of a fixed non-bipartite graph H. However, for a general bipartite graph H the problem is wide open. One of the most notorious problems in this direction is the Erdős-Sós conjecture from 1962, which determines the average degree forcing a copy of each tree T of a given size k. A solution of this conjecture for large k has been announced in the early 1990's by Ajtai, Komlós, Simonovits, and Szemerédi [1]. A trivial bound for the average degree guaranteeing containment of T is $2k$. Indeed, we can find a subgraph of minimum degree at least k and then embed T using the greedy procedure. A different approach to the problem is to relax the condition of minimum degree by investigating how many vertices of degree k guarantee the containment of a tree of order $k+1$. The Loebl-Komlós-Sós conjecture asserts that only half of the vertices need to have degree at least k. The conjecture has been solved exactly for large dense graphs [2,9] and proved to be asymptotically true for sparse graphs [4,5,6,7]. The Loebl-Komlós-Sós conjecture is best possible when we consider the class of all trees of order $k+1$, in particular, it is tight for paths. To observe this, consider a graph consisting of a disjoint union of copies of a graph H of order $k+1$ consisting of a clique of size $\lfloor \frac{k+1}{2} \rfloor - 1$, an independent set on the remaining vertices, and the complete bipartite graph between the two sets. Almost half of the vertices of this graph have degree k, but it does not contain a path on $k+1$ vertices as a subgraph.

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[2] Email: tereza@kam.mff.cuni.cz
[3] Email: piguet@cs.cas.cz
[4] Email: vaclavrozhon@gmail.com