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Decomposing graphs into paths and trees

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Abstract

In [4], the authors conjecture that for a fixed tree T, the edge set of any graph G of size divisible by size of T with sufficiently high degree can be decomposed into disjoint copies of T, provided that G is sufficiently highly connected in terms of maximal degree of T. In [4], the conjecture was proven for trees of maximal degree 2 (i.e., paths). In particular, it was shown that in the case of paths, the conjecture holds for 24-edge-connected graphs. We improve this result showing that 3-edge-connectivity suffices, which is best possible. We disprove the conjecture for trees of maximum degree greater than two and prove a relaxed version of the conjecture that concerns decomposing the edge set of a graph into disjoint copies of two fixed trees of coprime sizes.

Keywords: edge-decomposition, Barát-Thomassen conjecture, decomposition into paths

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1 Introduction and results

Graphs we consider are simple, the size of a graph is its number of edges. Given a tree T, we say that a graph has a *T-decomposition* if its edge set can be decomposed into disjoint copies of T. In [2], Barát and Thomassen conjectured that for a fixed tree T, the edge set of any sufficiently highly connected graph G of size divisible by size of T has a T-decomposition. After a series of partial results [1,4,5,6,7,8,9,10,11,12], the conjecture was recently proven in [3].

Theorem 1.1 *For any tree T, there exists an integer k_T such that every k_T-edge-connected graph of size divisible by size of T has a T-decomposition.*

Barát and Thomassen [2] also observed a correspondence between the existence of T-decompositions and nowhere zero flows. In particular, that the conjecture for T being a *claw*, that is $K_{1,3}$, is equivalent to Jaeger's conjecture, a weaker variant of Tutte's 3-flow conjecture, which asserts that there is an integer k such that every k-edge-connected graph admits a nowhere-zero 3-flow.

In [4], the authors posed the following, strengthened version of the conjecture of Barát and Thomassen.

Conjecture 1.2 *There is a function f such that, for any fixed tree T with maximum degree Δ_T, every $f(\Delta_T)$-edge-connected graph of size divisible by size of T with minimum degree at least $f(|E(T)|)$ has a T-decomposition.*

In [4], the conjecture was proven for trees of maximal degree 2, that is, paths.

Theorem 1.3 *For any path P, there exists d_P such that the edge set of every 24-edge-connected graph of size divisible by size of P with minimum degree d_P has a P-decomposition.*

We show that 24 in the statement of the theorem can be replaced by 3, which is best possible (as observed in [4]).

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