Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 60 (2017) 25–31
www.elsevier.com/locate/endm

Colourings of graphs by labellings

Martin Bača [1,2]

Department of Appl. Mathematics and Informatics
Technical University, Letná 9,
SK-042 00 Košice, Slovak Republic

Stanislav Jendrol' [1,3]

Institute of Mathematics
P.J. Šafárik University, Jesenná 5,
SK-041 54 Košice, Slovak Republic

Abstract

In this paper we give a survey on several types of colourings of elements of graphs by different types of labellings.

Keywords: irregularity strength, total vertex (edge) irregularity strength, irregular colouring, proper colouring.

1 Introduction

All graphs considered in this paper are simple and finite. We use the standard graph theory terminology. For notions not defined in this paper see the book

[1] This work was supported by the Slovak Science and Technology Assistance Agency under the contract No. APVV-15-0116 and Slovak VEGA grant 1/0368/16

[2] Email: martin.baca@tuke.sk

[3] Email: stanislav.jendrol@upjs.sk

http://dx.doi.org/10.1016/j.endm.2017.06.004
1571-0653/© 2017 Elsevier B.V. All rights reserved.

[10] of Bondy and Murty.

Let $G = (V, E)$ be a graph with vertex set V and edge set E. If $G = (V, E, F)$ is plane, then F denotes the set of faces of G. The set $V \cup E$ and the set $V \cup E \cup F$ is the set of *elements* of G. Let $X \in \{V, E\}$. For each element $x \in X$, let $S(x)$ be a nonempty subset of the set of elements of G and $\mathcal{S} = \{S(x)|x \in X\} = \{S(x)\}$. For positive integer k we consider a *labelling* of $\bigcup_{x\in X} S(x)$; this is a mapping l from $\bigcup_{x\in X} S(x)$ into the set of integers $\{1, \dots, k\}$.

Furthermore, we define the corresponding colouring c by $c(y)$ and $c(y) = \sum_{x\in S(y)} l(x)$ for $y \in X$. The colouring c is called *irregular* if $c(u) \neq c(v)$ for any two distinct elements u and v of X, and is called *proper*, if $c(u) \neq c(v)$ for any two adjacent elements u and v of G, unless $S(u) = S(v)$.

Moreover, for a vertex colouring and fixed $\mathcal{S}$, let $\chi_i(G, \mathcal{S})$ and $\chi_p(G, \mathcal{S})$ be the minimum k such that there exists a corresponding irregular colouring and the corresponding proper colouring, respectively.

Note that $\chi_i(G, \{\{v\}\}) = |V|$ and $\chi_p(G, \{\{v\}\}) = \chi(G)$, where $\chi(G)$ is the chromatic number of G, see [22].

Analogously, for an edge colouring and fixed $\mathcal{S}$, let $\chi'_i(G, \mathcal{S})$ and $\chi'_p(G, \mathcal{S})$ be the minimum k such that there exists a corresponding irregular colouring and the corresponding proper colouring, respectively. Note that $\chi'_i(G, \{\{e\}\}) = |E|$ and $\chi'_p(G, \{\{e\}\}) = \chi'(G)$, where $\chi'(G)$ is the chromatic index of G.

2 Colourings by edge labellings

Let $N_E(v)$ denote the set of edges incident with $v \in V$. In 1988 Chartrand et al. [13] initiated a study of the parameter $\chi_i(G, \{N_E(v)\})$, which is called the *irregularity strength* of a graph G having no component K_2 and at most one K_1.

Aigner and Triesch [3] proved that $\chi_i(G, \{N_E(v)\}) \leq p-1$ if G is connected graph of order p, $G \neq K_3$, and $\chi_i(G, \{N_E(v)\}) \leq p + 1$ otherwise. Nierhoff [28] showed that this parameter is at most $p - 1$ for all graphs distinct from K_3. The bound is tight e.g. for stars. For very nice surveys on this parameter see Lehel [26], a contribution by Frieze, Gould, Karoński, and Pfender [17], and a recent paper by Cuckler and Lazebnik [14]. Valuable contribution to this topic have been done by Przybyło [30,31].

Karoński, Łuczak, and Thomason posed the following well known and popular 1-2-3-conjecture:

Download English Version:

https://daneshyari.com/en/article/5777152

Download Persian Version:

https://daneshyari.com/article/5777152

Daneshyari.com