Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 60 (2017) 39–46

www.elsevier.com/locate/endm

On Algebraic Expressions of Directed Grid Graphs

Mark Korenblit [1]

Department of Computer Science
Holon Institute of Technology
Holon, Israel

Abstract

The paper investigates relationship between algebraic expressions and labeled graphs. We consider directed grid graphs having m rows and n columns. Our intent is to simplify the expressions of these graphs. With that end in view, we describe two algorithms which generate expressions of polynomial sizes for directed grid graphs.

Keywords: Labeled graph, two-terminal directed acyclic graph, series-parallel graph, grid, expression, backtracking, decomposition.

1 Introduction

A *graph* $G = (V, E)$ consists of a *vertex set* V and an *edge set* E, where each edge corresponds to a pair (v, w) of vertices. If the edges are ordered pairs of vertices (i.e., the pair (v, w) is different from the pair (w, v)), then we call the graph *directed* or *digraph*; otherwise, we call it *undirected*. If (v, w) is an edge in a digraph, we say that (v, w) *leaves* vertex v and *enters* vertex w. A vertex in a digraph is a *source* if no edges enter it, and a *sink* if no edges leave it.

[1] Email: `korenblit@hit.ac.il`

http://dx.doi.org/10.1016/j.endm.2017.06.006
1571-0653/© 2017 Elsevier B.V. All rights reserved.

A two-terminal directed acyclic graph (*st-dag* in [1]) has only one source and only one sink.

Given a graph $G = (V, E)$, an *edge labeling* is a function $E \longrightarrow R$, where R is a ring equipped with two binary operations $+$ (addition or disjoint union) and $\cdot$ (multiplication or concatenation, also denoted by juxtaposition when no ambiguity arises). In what follows, elements of R are called *labels*, and a *labeled graph* refers to an edge-labeled graph with all labels distinct.

A path between the source and the sink of an st-dag is called *spanning.* We define the sum of edge label products corresponding to all possible spanning paths of an st-dag G as the *canonical expression* of G. The label order in every product (from the left to the right) is identical to the order of corresponding edges in the path (from the source to the sink). An algebraic expression is called an *st-dag expression* (a *factoring of an st-dag* in [1]) if it is algebraically equivalent to the canonical expression of an st-dag. An st-dag expression consists of labels, the two ring operators $+$ and $\cdot$, and parentheses. We denote an expression of G by $Ex(G)$.

We define the total number of labels in an algebraic expression as its *complexity.* An *optimal representation of the algebraic expression* F is an expression of minimum complexity algebraically equivalent to F. We consider expressions with a minimum (or, at least, a polynomial) complexity as a key to generating efficient algorithms on distributed systems.

A *series-parallel graph* is defined recursively so that a single edge is a series-parallel graph and a graph obtained by a parallel or a series composition of series-parallel graphs is series-parallel. A series-parallel graph expression has a representation in which each label appears only once [1], [7] (*read-once formula* [4]). This representation is an optimal representation of the series-parallel graph expression.

As shown in [2], an st-dag is series-parallel if and only if it does not contain a subgraph which is a homeomorph of the *forbidden subgraph* consisting of edges e_1, e_2, e_3, b_1, c_1 of the graph illustrated in Fig. 1. Possible optimal representations of its expression are $e_1\left(e_2e_3 + c_1\right) + b_1e_3$ or $\left(e_1e_2 + b_1\right)e_3 + e_1c_1$. For this reason, an expression of a non-series-parallel st-dag can not be represented as a read-once formula. However, for arbitrary functions, generating the optimum factored form is NP-complete [12].

Problems related to computations on labeled graphs have applications in various areas. Specifically, flow [11], scheduling [3], reliability [10], economical [9] problems which are either intractable or have complicated solutions in the general case are solvable for series-parallel graphs.

In [7] we presented an algorithm generating the expression of $O\left(n^2\right)$ com-

Download English Version:

https://daneshyari.com/en/article/5777154

Download Persian Version:

https://daneshyari.com/article/5777154

Daneshyari.com