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On regular handicap graphs of even order

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Abstract

Let $G = (V, E)$ be a simple graph of order n. A bijection $f : V \to \{1, 2, \ldots, n\}$ is a *handicap labeling* of G if there exists an integer ℓ such that $\sum_{u \in N(v)} f(u) = \ell + f(v)$ for all $v \in V$, where $N(v)$ is the set of all vertices adjacent to v. Any graph which admits a handicap labeling is a *handicap graph*.

We present an overview of results, which completely answer the question of existence of regular handicap graphs of even order.

Keywords: Graph labeling, handicap labeling, regular graphs
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1 Introduction and definitions

Let $G = (V, E)$ be a simple graph of order n. A bijection $f : V \to \{1, 2, \ldots, n\}$ is a *handicap labeling* of G if there exists an integer ℓ such that $\sum_{u \in N(v)} f(u) = \ell + f(v)$ for all $v \in V$, where $N(v)$ is the set of all vertices adjacent to v. Any graph which admits a handicap labeling is a *handicap graph*.

Handicap labelings were introduced as a modification of a distance magic labeling, which is a bijection $f : V \to \{1, 2, \ldots, n\}$ with the property that the sum $\sum_{u \in N(v)} f(u)$ equals the same value for every $v \in V$. The motivation of both labelings lies in scheduling of incomplete tournaments with teams ordered linearly according to their strength. The label $f(i)$ represents the rank that decreases with the team strength. We identify vertex names with their labels, thus by i we understand the vertex labeled i. A distance magic labeling of a graph represents a schedule of an incomplete tournament in which all teams should have an equally strong set of opponents, while in a handicap tournament a certain advantage is given to weaker teams: the weaker the team, the bigger its advantage. This hopes to support attractive tournaments in which each game counts. An excellent up-to-date overview of recent results on labelings is the review by Gallian [6], a specialized survey on distance magic labelings and its application to tournaments is [1].

For any graph with given regularity and given order an easy counting argument shows (see [7]) the set of vertex weights is given by the following lemma, unlike vertex-magic total labelings, where for the same graph different weights using the same set of labels can be obtained.

Lemma 1.1 *In an r-regular handicap graph with n vertices the weight of every vertex is $w(i) = (r-1)(n+1)/2 + i$.*

Each vertex weight is an integer value obtained as a sum of integers, thus

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