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A New Graph Construction of Unbounded Clique-width

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Abstract

We define *permutation-partition graphs* by replacing one part of a $2K_2$-free bipartite graph (a bipartite *chain* graph) by an induced linear forest. We show that this hereditary graph class is of of unbounded clique-width (with a new graph construction of large clique-width). We show that this graph class contains no minimal graph class of unbounded clique-width, and give a conjecture for a contained *boundary class* for this property.

Keywords: hereditary graph classes, clique-width

1 Introduction

We start by introducing the notion of *clique-width*. The *clique-width* of a graph was first introduced in [5] and is defined as the minimum number of labels needed to construct the graph by means of the four graph operations:

(i) creation of a new vertex v with label i (denoted $i(v)$);

(ii) disjoint union of two labeled graphs G and H (denoted $G \oplus H$);

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(iii) connecting vertices with specified labels i and j (denoted $\eta_{i,j}$);

(iv) renaming label i to label j (denoted $\rho_{i\to j}$)

The clique-width of a graph G will be denoted $\mathrm{cwd}(G)$.

Every graph can be defined by an algebraic expression using the four operations above. This expression will be called a k-expression if it uses k different labels. For instance, the cycle C_5 on vertices a, b, c, d, e (listed along the cycle) can be defined by the following 4-expression:

$$\eta_{4,1}(\eta_{4,3}(4(e) \oplus \rho_{4\to 3}(\rho_{3\to 2}(\eta_{4,3}(4(d) \oplus \eta_{3,2}(3(c) \oplus \eta_{2,1}(2(b) \oplus 1(a)))))))).$$

Alternatively, any algebraic expression defining G can be represented as a rooted tree, called a *parse tree*, whose leaves correspond to the operations of vertex creation, the internal nodes correspond to the $\oplus$-operations, and the root is associated with G. The operations η and ρ are assigned to the respective edges of the tree. Figure 1 shows the tree representing the above expression defining a C_5.

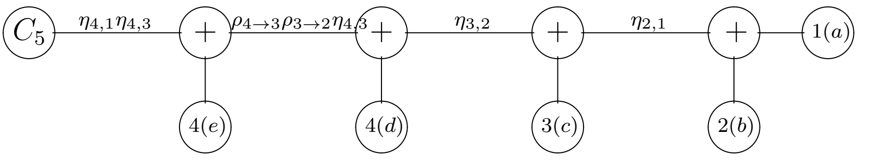

Fig. 1. The tree representing the expression defining a C_5

Clique-width is a relatively new notion compared to another important graph parameter, *tree-width*. The notion of clique-width generalizes that of tree-width in the sense that graphs of bounded tree-width have bounded clique-width.

The importance of these graph invariants is due to the fact that numerous problems that are NP-hard in general admit polynomial-time solutions when restricted to graphs of bounded tree- or clique-width (see e.g. [2,6]). The celebrated result due to Robertson and Seymour states that for any planar graph H there is an integer N such that the tree-width of graphs containing no H as a minor is at most N [12]. In other words, the planar graphs constitute a unique minimal minor-closed class of graphs of unbounded tree-width. A special role in this class is assigned to rectangle grids, because every planar graph is a minor of some large enough grid and grids can have arbitrarily large tree-width. Therefore, grids form the only "unavoidable minors" in graphs of