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Characterizations of generalized exponential trichotomies for linear discrete-time systems

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Abstract

The generalized exponential trichotomy (g.e.t) concept is explored for linear time-varying systems. Characterizations and relations with the notion of uniform exponential trichotomy (u.e.t) in the sense of Elaydi-Janglajew are also provided.

Keywords: difference equations, (generalized) exponential trichotomy, discrete-time linear time-varying systems.

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1 Introduction

The trichotomy concept represents a complex description of the asymptotic behavior of linear time-varying (LTV) systems, involving the partition of the state space, at all times, into three subspaces (stable, unstable and central).

The first notable study on the uniform exponential trichotomy (u.e.t) for discrete-time LTV systems was done by S. Elaydi and K. Janglajew in [2]. This study has been extended in many directions. For example, various nonuniform exponential concepts are presented in [4], [7], [8] (and the references therein), where some exponential loss of hyperbolicity along the trajectories is allowed. Also, nonuniform concepts of polynomial type are presented in [9]. It should be noted that our g.e.t. notion is a kind of uniform hyperbolicity. For some important results obtained in this direction, see [1], [3].

In this paper we give a simple but suggestive example illustrating the relationship between the u.e.t and g.e.t concepts. Also, motivated by the lead given in [2], we present some theorems of characterizations for discrete-time LTV systems in terms of g.e.t.

2 Generalized exponential trichotomy

Throughout the paper $\mathbb{Z}$ denotes the set of real integers, $\mathbb{Z}_+$ is the set of all $n \in \mathbb{Z}$, $n \geq 0$, and $\mathbb{Z}_-$ is the set of all $n \in \mathbb{Z}$, $n \leq 0$. Moreover, by $\mathbb{R}$ we will denote the set of real numbers and $\|.\|$ represents a matrix norm.

Consider the LTV system

$$x_{n+1} = A_n x_n, \quad n \in \mathbb{Z} \tag{$\mathfrak{A}$}$$

where terms $(A_n)_{n\in\mathbb{Z}}$ are $n \times n$ invertible matrices. By $W(n)$ is denoted the fundamental matrix of ($\mathfrak{A}$), i.e. $W_{n+1} = A_n W_n$ and $W(0) = I$, where I is the unit matrix. Further, consider the strictly positive sequence $\{a_n\}_{n\in\mathbb{Z}}$ satisfying

$$\sum_{j=p}^{q} a_j \to +\infty \;\text{ as }\; q \to +\infty \;\text{ for fixed }\; p \in \mathbb{Z}; \tag{1}$$

$$\sum_{j=p}^{q} a_j \to +\infty \;\text{ as }\; p \to -\infty \;\text{ for fixed }\; q \in \mathbb{Z}; \tag{2}$$

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