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ScienceDirect
Electronic Notes in Discrete Mathematics 59 (2017) 3–17
Electronic Notes in DISCRETE MATHEMATICS
www.elsevier.com/locate/endm

Toward uniform random generation in 1-safe Petri nets

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Abstract

We study the notion of uniform measure on the space of infinite executions of a 1-safe Petri net. Here, executions of 1-safe Petri nets are understood up to commutation of concurrent transitions, which introduces a challenge compared to usual transition systems. We obtain that the random generation of infinite executions reduces to the simulation of a finite state Markov chain. Algorithmic issues are discussed.

Keywords: Petri net, partial order model, uniform generation.

1 Introduction

Petri nets are formal models designed to describe and analyze the behavior of concurrent systems. Among the many kinds of systems where Petri nets may be introduced to formally describe a concurrent dynamics, distributed databases [9] and telecommunication networks [5] are two typical examples. Both examples involve temporal evolution on the one hand, and the paradigm

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http://dx.doi.org/10.1016/j.endm.2017.05.002

of resource sharing on the other hand, where resources are "spatially" distributed. Requests for resources are local, in such a way that any two actions requiring disjoint sets of resources may be considered as parallel.

Since their initial introduction in the 1960's, several variants of Petri nets have been studied. In this paper, we shall limit ourselves to 1-safe Petri nets, which we briefly define now. An *unmarked Petri net* is a triple $N = (P, T, F)$, where P and T are two finite and disjoint non empty sets of *places* and of *transitions* respectively, and $F \subseteq (P \times T) \cup (T \times P)$ is called the *flow relation*. Graphically, places are traditionally represented by circles and transitions are represented by squares or rectangles (see Figure 1). The flow relation is depicted by arrows from places to transitions and from transitions to places.

Given a transition $t \in T$, the *preset* ${}^\bullet t$ and the *postset* $t^\bullet$ of t are the sets of places defined as follows:

$$ {}^\bullet t = \{p \in P \ : \ (p, t) \in F\}, \qquad t^\bullet = \{p \in P \ : \ (t, p) \in F\}. $$

It is assumed that the preset and the postset of any transition are both nonempty.

A *marking* of N is any integer-valued function $M : P \to \mathbb{N}$. The marking is said to be 1-*safe*, or simply *safe*, whenever $M(p) \leq 1$ for all places $p \in P$. The number $M(p)$ is interpreted as a number of *tokens* lying in the place p. Tokens are graphically represented inside places, as in Figure 1. Given a marking M of N, and a transition $t \in T$, we say that t can *fire from* M, or that M *enables* t, whenever $M(\cdot) > 0$ on ${}^\bullet t$.

If t can fire from M, then the *firing rule* $M \xrightarrow{t} M'$ defines the new marking M' as follows (see the commentary below and the illustration in Figure 1):

$$ \text{for all } p \in P, \quad M'(p) = \begin{cases} M(p), & \text{if } p \notin ({}^\bullet t \cup t^\bullet) \\ M(p) - 1, & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + 1, & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p), & \text{if } p \in ({}^\bullet t \cap t^\bullet) \end{cases} $$

The intuitive interpretation of the firing rule is as follows. The tokens located in places of the preset ${}^\bullet t$ are *resources*, needed for firing and consumed by the transition when fired; this explains the second rule. The firing of t also produces new resources, *i.e.*, tokens; the third rule specifies that the new tokens are created in the postset of t. If a place belongs to both the pre- and the postset of t, then we can see the fourth rule as the simultaneous instance