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Extended boxed product and application to synchronized trees

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Abstract

We introduce a new technique to specify increasing labeled structures. For such objects, the boxed product as introduced by Greene is sufficient to efficiently specify the class, however for particular classes the size of the specification is very large. In particular, in the case of partially ordered sets, the calculus of the total orders compatibles with the poset (called linear extensions) can be tedious. We here developed an idea due to Stanley that uses a geometrical interpretation to calculate the linear extensions of a given poset. We will present a way to extend this idea to the

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symbolic method, and illustrate it with the example of specific increasing trees with exactly one repeated label, and show how to uniformly generate such structures.

Keywords: synchronized tree, increasing tree, uniform random sampling, recursive method, analytic combinatorics.

1 Introduction

In 1983, Greene [9] developed new approaches to address labeled combinatorial structures that require some order constraints for the labeling. In particular, he introduced the binary boxed operator $\mathcal{A}^{\Box} \star \mathcal{B}$ that encodes the labeled product of the classes $\mathcal{A}$ and $\mathcal{B}$ satisfying the fact that the smallest label appears in the component of $\mathcal{A}$. From the standard convolution issued in the product of the related exponential generating functions $A(z)$ and $B(z)$, Greene deduced that the generating function for $\mathcal{A}^{\Box} \star \mathcal{B}$ is given by

$$\int_0^z A'(t) \cdot B(t) dt.$$

This now classical constrained product has been fully integrated into the *symbolic method* (cf. [7]) and many structures are based on this simple order constraint. For instance, specific subclasses of permutations (cf. several examples and references in [7, II.6.3.]), increasingly labeled trees [1] or partially increasingly labeled trees [2] can be specified thanks to the boxed product.

In the random sampling domain, the boxed product also appears as a cornerstone for the efficiency of the recursive method [8] [10]. Although other elegant specifications based on such ideas are presented by Flajolet and Sedgewick, it seems clear that some more intricate problems cannot be encoded with only such a global order constraint. The class of increasingly labeled directed acyclic graphs contains much more complex dependencies than we can expect to simply encode with only boxed products.

More specifically, if we consider two directed sequences of atoms, add a top node that is the common ancestor of both sequences and a bottom node that is the common descendant of both sequences, the boxed product seems to be inappropriate in order to simply describe an increasing labeling of such a structure. Even if these structures can in fact be specified by using boxed

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