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Scheduling parallel jobs on heterogeneous platforms

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Abstract

We consider the problem of scheduling parallel jobs on heterogeneous platforms. Given a set $\mathcal{J}$ of n jobs where each job $j \in \mathcal{J}$ is described by a pair (p_j, q_j) with a processing time p_j and number q_j of processors required and a set of N heterogeneous platforms P_i with m_i processors, the goal is to find a schedule for all jobs on the platforms minimizing the maximum completion time. The problem is directly related to a two-dimensional multi strip packing problem. Unless $P = NP$ there is no approximation algorithm with absolute ratio better than 2 for the problem. We propose an approximation algorithm with absolute ratio 2 improving the previously best known approximation algorithms. This closes the gap between the lower bound of < 2 and the best approximation ratio.

Keywords: scheduling parallel tasks, strip packing, approximation algorithms.

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1 Introduction

We study the problem of scheduling parallel jobs on heterogeneous platforms. The input consists of a set $\mathcal{J} = \{1, \ldots, n\}$ of n jobs and a set $\mathcal{B}$ of N platforms $P_1, \ldots, P_N$, where each P_i consists of a set $M_i = \{1, \ldots, m_i\}$ of processors for $i \in [N] := \{1, \ldots, N\}$. The width of the platform P_i is the number m_i of processors. Each job $j \in \mathcal{J}$ is described by a pair (p_j, q_j) with a processing time (or height) $p_j \in \mathbb{N}$ and number of processors (or width) $q_j \in \mathbb{N}$ required to execute j. If all numbers m_i are equal, we have identical platforms. In the general case the numbers m_i may be different and the machines are called heterogeneous platforms. For simplification we suppose that $m_1 \geq m_2 \geq \ldots \geq m_N$. A schedule is an assignment $a : \mathcal{J} \to \mathbb{Q}_{\geq 0} \times \cup_{i=1}^{N} 2^{M_i}$ that assigns every job j to a starting time $t_j = a_1(j)$ and to a subset $A_j = a_2(j) \subseteq M_i$ of processors of a platform P_i such that $|A_j| = q_j$. A job j can only be executed in platform P_i if the width of the platform $m_i \geq q_j$. A schedule is feasible if every processor in every platform executes at most one job at any time. The goal is to find a feasible schedule with minimum total length or makespan $\max_{i \in [N]} C_{max}(P_i)$ where $C_{max}(P_i) = \max_{j | A_j \subseteq M_i} t_j + p_j$ is the local makespan on platform P_i (or height of platform P_i). The optimum value for an instance $(\mathcal{J}, \mathcal{B})$ is denoted by $OPT(\mathcal{J}, \mathcal{B})$.

2 Previous and new Results

Table 1
Approximation algorithms for heterogeneous platforms.

		ratio	constraints
Tchernykh et al. [6]	2005	10	none
Schwiegelshohn et al. [5]	2008	3	non-clairvoyant
Tchernykh et al. [7]	2010	$2e + 1$	release dates
Bougeret et al. [1]	2010	2.5	$\max q_j \leq \min m_i$
Dutot et al. [2]	2013	$(2 + \epsilon)$	none
Jansen and Trystram **(new result)**	2016	2	none

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