Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 55 (2016) 13–16

www.elsevier.com/locate/endm

Experiments with two heuristic algorithms for the Maximum Algebraic Connectivity Augmentation Problem

Claudia Justel [a,1] Carlos Rocha [a,2] and Emanuelle Chaves [a,3]

[a] *Instituto Militar de Engenharia, Rio de Janeiro, Brasil*

Abstract

In this work we present a heuristic algorithm to solve the Maximum Algebraic Connectivity Augmentation Problem (MACAP). This is an NP-complete problem (proved by Mosk-Aoyama in 2008) and consists in, given a graph, determining the smallest set of edges not belonging to it in such a way that the value of the algebraic connectivity of the augmented graph is maximum. In 2006, Ghosh and Boyd presented a heuristic procedure to solve this problem. This heuristic is an iterative method that selects one edge at a time based on the values of the components of a Fiedler vector of the graph. Our goal is to increase the value of the algebraic connectivity of a given graph by inserting edges based on the eccentricity of vertices. In order to evaluate our algorithm, computational tests comparing it with the Ghosh and Boyd procedure are presented.

Keywords: Graph, Laplacian matrix, Algebraic connectivity, Approximated algorithm.

[1] Email: `cjustel@ime.eb.br`, speaker

[2] Email: `edugaspar@hotmail.com`

[3] Email: `manuenlc@gmail.com`

http://dx.doi.org/10.1016/j.endm.2016.10.004

1 Introduction

The algebraic connectivity, defined as the second smallest eigenvalue of the Laplacian matrix of a graph G, is a spectral invariant widely studied in the literature. This parameter is related to the connectivity of the graph. There are different applications of this parameter in several problems ([4])[7],[8]). In this work, we deal with a NP-complete problem known as the Maximum Algebraic Connectivity Augmentation (MACAP). We present a heuristic procedure, which is an iterative method where edges are added depending on the values of the eccentricities of its endpoints. This strategy is different from the heuristic procedure proposed by Ghosh and Boyd in 2006 [2]. The rest of the paper is organized as follows: in Section 2, the definition of the MACAP and the heuristic algorithm proposed by Ghosh and Boyd are given. Section 3 presents a new heuristic algorithm and experimental results comparing this new algorithm with the approximate algorithm in Section 2. At last, final remarks are presented in Section 4. Basic concepts and notation in Graph and Spectral Graph Theory can be found in [1] and [3].

2 MACAP: Complexity, Ghosh and Boyd's heuristic

Given a graph $G = (V, E)$ and a non-negative integer k, the MACAP consists in determining, among all the subsets of edges in G^C of size at most k, the subset that increases the algebraic connectivity as much as possible. In [4] Mosk-Aoyama proved that the decision problem associated to the MACAP is NP-Complete. The heuristic procedure presented in [2] by Ghosh and Boyd, uses a Fiedler vector in order to determine a set of edges to be included in the input graph G to increase the value of the algebraic connectivity. The notation used is: $G_{base} = (V, E_{base})$ a graph with $|V| = n$, $E_{cand} \subset E^C_{base}$ a subset of candidate edges of size m_c, and a non-negative integer number k, $0 \leq k \leq m_c$. The heuristic, denoted Perturbation Heuristic, PH, chooses k edges in E_{cand} ($E \subseteq E_{cand}$, $|E| = k$) to be inserted in G_{base}. Let $L = L(G)$ be the Laplacian matrix of G, $\lambda_2(G)$ the algebraic connectivity of G and $\mathbf{w} = (w_1, \ldots, w_n)$ a Fiedler vector, an eigenvector associated to $\lambda_2(G)$. The greedy strategy used selects k edges, one at a time, being the edge $(i, j) \in E^C_{base}$ for which the components i and j of the Fiedler vector $\mathbf{w}$ has the greatest value of $(w_i - w_j)^2$. In this case, a set of candidates edges E_{cand} that may be different than the set E^C is considered. Ghosh and Boyd ([2]) discuss the results obtained with their algorithm for 3 types of graphs (randomly generated) and suggest that "a large increase in algebraic connectivity can be obtained by adding a few edges carefully". This suggestion is what motivates our work.