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# Stochastic geometric programming with joint probabilistic constraints

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**Abstract**

This paper discusses the geometric programs with joint probabilistic constraints. When the coefficients are normally distributed and independent of each other, we approximate the problem by using piecewise linear function and transform the approximation problem into a geometric program. We prove that this approximation method provides a lower bound, and we use Bonferroni approximation to find an upper bound.

*Keywords:* Geometric programs; Joint probabilistic constraints; Piecewise linear approximation; Bonferroni approximation.

## 1 Introduction

Geometric programs are a type of optimization problems characterized by an objective and constraints functions which have a special form [2]. In real world

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applications, some of the coefficients in a geometric program may not be known precisely when the optimization is made. Hence, the stochastic geometric programming models are proposed to model geometric problems with random parameters. Individual probabilistic constraints have been applied to control the uncertainty level of geometric constraints [4,5,7].

In this paper, we furthermore consider the following joint probabilistic constrained stochastic geometric programs:

$$\min_t E\left[\sum_{i\in I_0} c_i \prod_{j=1}^{M} t_j^{a_{ij}}\right] \tag{1}$$

$$\text{s.t. } P\left(\sum_{i\in I_k} c_i \prod_{j=1}^{M} t_j^{a_{ij}} \leq 1,\ k=1,\cdots,K\right) \geq 1-\epsilon. \tag{2}$$

Here, $\{I_k,\ k=0,\cdots,K\}$ is a decomposition of $\{1,\cdots,Q\}$ into $K+1$ disjoint index sets. $Q$ is the total number of monomilas $c_i\prod_{j=1}^{M} t_j^{a_{ij}}$ in (1) and (2). Unlike [4,5,7], we require that the overall probability of meeting the $K$ geometric constraints is above a certain probability level $1-\epsilon$, $\epsilon\in(0,0.5]$.

The stochastic geometric program with joint probabilistic constraints is a special kind of joint probabilistic constrained problems. The linear programs with joint probabilistic constraints are widely studied in [1,3,6].

## 2 Approximation methods

Similarly to [4], we suppose that $a_{ij}$ is deterministic and $c_i$ is normally distributed and independent of each other, i.e., $c_i \sim N(E_{c_i}, \sigma_i^2)$.

As $c_i$ are independent, (2) is equivalent to

$$\prod_{k=1}^{K} P(\sum_{i\in I_k} c_i \prod_{j=1}^{M} t_j^{a_{ij}} \leq 1) \geq 1-\epsilon. \tag{3}$$

By introducing auxiliary variables $y_k\in\mathbb{R},\ k=1,\cdots,K$, (3) can be equivalently transformed into

$$P(\sum_{i\in I_k} c_i \prod_{j=1}^{M} t_j^{a_{ij}} \leq 1) \geq y_k,\ k=1,\cdots,K, \tag{4}$$

and

$$\prod_{k=1}^{K} y_k \geq 1-\epsilon,\ y_k\geq 0. \tag{5}$$