Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 55 (2016) 61–64

www.elsevier.com/locate/endm

Column generation for the variable cost and size bin packing problem with fragmentation

Marco Casazza [1]

UPMC University Paris 06, UMR 7606, LIP6, F-75005, Paris, France

Alberto Ceselli [2]

Dipartimento di Informatica, Università Degli Studi di Milano, Crema, Italy

Abstract

Bin Packing Problems with Item Fragmentation (BPPIF) are variants of classical Bin Packing in which items can be split at a price. We extend BPPIF models from the literature by allowing a set of heterogeneous bins, each potentially having a different cost and capacity. We introduce extended formulations and column generation algorithms to obtain good bounds with reasonable computing effort. We test our algorithms on instances from the literature. Our experiments prove our approach to be more effective than state-of-the-art general purpose solvers.

Keywords: Bin Packing, Item Fragmentation, Variable Cost and Size, Column Generation.

1 Introduction

Bin Packing Problems with Item Fragmentation (BPPIF) haves been introduced to model problems in diverse domains, like routing of consolidated traffic in optical networks and VLSI circuit design [1]. In their bin-minimization variant a set I of items, each having a size d_i, and a set of bins J, each having a capacity C, are given, together with a *fragmentation budget* F. The aim is to assign items to the minimum number of bins; up to F item splits are

[1] Email: marco.casazza@upmc.fr

[2] Email: alberto.ceselli@unimi.it, partially funded by Regione Lombardia - Fondazione Cariplo, grant n. 2015-0717, project “REDNEAT”

http://dx.doi.org/10.1016/j.endm.2016.10.016
1571-0653/

allowed: whenever an item is split, it is replaced by two fragments; the split point is arbitrary, but the sum of fragment sizes must equal the size of the original item. Recursive fragmentations are allowed, but each split counts in the budget. The final set of fragments need then to be assigned to the bins, in such a way that the sum of item (fragment) sizes assigned to the same bin do not exceed C. Recent contributions to the field include both approximation algorithms [3] and exact methods [2], both approaches proving to be effective. As stressed in [3], major interest is currently in making BPPIF models more flexible. In this paper we tackle the generalization of BPPIF, in its bin-minimization variant, in which each bin $j \in J$ has a potentially different cost v_j and capacity c_j, and the overall cost of the used bins needs to be minimized. We refer to our generalization as the Variable Cost and Size BPPIF (VCSB).

2 Model

We first observe the following.

Proposition 2.1 *An optimal VCSB solution always exists, in which (a) each item is split in at most two fragments (b) each bin contains at most two fragmented items (c) each set of k bins contains at most $k-1$ fragmented items.*

Any solution satisfying (a)–(c) is called *primitive* [1]. A formal proof is omitted, but intuitively given a set of k bins and a solution assigning a subset of items $\bar{I} \subseteq I$ to them, a *Next Fit with Fragmentations* procedure produces a fragmentation pattern that comply with (a)–(c). Fragmented items link one bin another in a *chain* structure, that includes a subset $\bar{I} \subseteq I$ of items and a subset $\bar{J} \subseteq J$ of bins. On feasible chains it always holds $\sum_{i \in \bar{I}} d_i \leq \sum_{j \in \bar{J}} c_j$. Let Ω be the set of all feasible chains. Following the framework of [2] we model the VSCB with the following chain-based extended formulation:

$$\min \sum_{p \in \Omega} \Big(\sum_{j \in J} v_j \bar{y}^p_j\Big) z^p \tag{1}$$

$$\text{s.t.} \quad \sum_{p \in \Omega} \bar{x}^p_i z^p = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{p \in \Omega} \bar{y}^p_j z^p \leq 1 \qquad \forall j \in J \tag{3}$$

$$\sum_{p \in \Omega} \bar{f}^p z^p \leq F \tag{4}$$

$$z^p \in \{0,1\} \qquad \forall p \in \Omega. \tag{5}$$