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Heuristics for the General Multiple Non-linear Knapsack Problem

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Abstract

We propose heuristic algorithms for the multiple non-linear knapsack problem with separable non-convex profit and weight functions. First, we design a fast constructive algorithm that provides good initial solutions. Secondly, we improve the quality of these solutions through local search procedures. We compare the proposed methods with exact and heuristic algorithms for mixed integer non-linear programming problems, proving that our approach provides good-quality solutions in smaller CPU time.

Keywords: Multiple non-linear knapsack problem, Heuristic algorithm, Local search, Mixed-integer non-linear programming.

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1 Introduction

In the multiple non-linear knapsack problem, we are given n items and m knapsacks. We aim at deciding how many units of item j to load in knapsack i, i.e., our decision variables are represented by $x_{ij} \geq 0$ for each $i = 1, \ldots, m$, $j = 1, \ldots, m$. The units of some items are indivisible, thus integrality requirements on the corresponding x_{ij} $(i = 1, \ldots, m)$ have to be satisfied. For each item j, we have

- an upper bound on the item availability $u_j > 0$;
- a profit function $f_j(x) : \mathbb{R}_+ \to \mathbb{R}_+$;
- a weight function $g_j(x) : \mathbb{R}_+ \to \mathbb{R}_+$.

We assume that $f(x)$ and $g(x)$ are twice continuously differentiable, separable, non-linear, non-negative, non-decreasing functions. Note that there is no further assumption, thus, in general, f and g can be non-convex and non-concave.

The *Multiple Non-Linear Knapsack Problem* (MNLKP) can then be written as:

$$\max \sum_{i \in M} \sum_{j \in N} f_j(x_{ij}) \tag{1}$$

$$\text{s.t.} \sum_{j \in N} g_j(x_{ij}) \leq c_i \qquad i \in M \tag{2}$$

$$\sum_{i \in M} x_{ij} \leq u_j \qquad j \in N \tag{3}$$

$$x_{ij} \geq 0 \qquad i \in M,\ j \in N \tag{4}$$

$$x_{ij} \text{ integer} \qquad i \in M,\ j \in \overline{N} \subseteq N \tag{5}$$

where $M = \{1, \ldots, m\}$ and $N = \{1, \ldots, n\}$. Objective function (1) aims at maximizing the profit given by the total amount of items inserted in the knapsacks. Constraints (2) impose that the knapsack maximum capacities are respected. The limit on the maximum availability of each item is represented by constraints (3). Constraints (5) ensure that, for the indivisible items, a discrete quantity is selected.

To the best of our knowledge, no author studied such variant of the non-linear knapsack before. For an extended reference on the classical 0-1 multiple knapsack the reader is referred to [6,5]. For a study on the single non-linear knapsack problem, we refer the reader to [3].