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ScienceDirect
Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 55 (2016) 73–76
www.elsevier.com/locate/endm

Dual approaches for a specific class of integer nonlinear programming problems

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Abstract

In this work, we propose a strategy for computing valid lower bounds for a specific class of integer nonlinear programming problems, that includes integer quadratic programming problems. This strategy is used within a branch-and-bound scheme. Experimental results for randomly generated instances show that, in the quadratic case, the devised branch-and-bound method compares favorably to the MIQP solver of `CPLEX 12.6` when the number of constraints is small.

Keywords: integer programming, quadratic programming, global optimization

We consider integer optimization problems of the following form:

$$\begin{aligned} \min\ & f(x) = (x^\top Q x)^p + L^\top x \\ \text{s.t.}\ & Ax \le b \\ & x_i \in \mathbb{Z},\ i = 1, \dots, n \end{aligned} \tag{1}$$

where $Q \in \mathbb{R}^{n\times n}$ is a positive definite matrix, $L \in \mathbb{R}^n$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$ and $0.5 < p \le 1$.

The motivation for studying this class of problems is twofold. From a practical point of view, Problem (1) includes problems that arise in applications, such as portfolio optimization problems (see e.g. [1]). From a theoretical point of view, defining effective algorithms to solve to global optimality Problem (1) represents a big challenge in itself.

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http://dx.doi.org/10.1016/j.endm.2016.10.019
1571-0653/

In this work, following what has been done in recent papers by Buchheim et al. (see e.g. [3], [4]), we propose a strategy for computing valid lower bounds of Problem (1), with the idea of using this strategy within a branch-and-bound scheme for MINLP problems.

The branch-and-bound scheme we consider enumerates nodes very quickly: by fixing the branching order in advance, we gain the advantage of shifting expensive computations into a preprocessing phase. In each node, the dual problem of the continuous relaxation is solved in order to determine a local lower bound. Since all constraints of the continuous relaxation of (1) are affine, strong duality holds if the primal problem is feasible.

More precisely, assume that the variables with indices in $I \subseteq \{0, \ldots, n\}$ have been fixed to values $s = (s_i)_{i \in I}$. Then, Problem (1) reduces to the minimization of

$$f_s : \mathbb{Z}^{n-|I|} \to \mathbb{R},\ x \mapsto (x^\top Q_s x + c_s^\top x + d_s)^p + L_s x + e_s \tag{2}$$

over the feasible region $\mathcal{F}_s = \{x \in \mathbb{Z}^{n-|I|} \mid A_s x \leq b_s\}$, where the matrix Q_s is obtained by deleting the rows and columns corresponding to I, the matrix A_s is obtained by deleting the columns corresponding to I, and the remaining terms are updated appropriately.

Let $\mathscr{L}_s(x, \lambda) : \mathbb{R}^{n-|I|} \times \mathbb{R}^m \to \mathbb{R}$ be the Lagrangian function associated to the continuous relaxation at a generic node. In Section 1 we show how to compute, for fixed λ, the unconstrained minimizer of the Lagrangian function, so that the dual problem we end up with is a continuous problem with non-negativity constraints:

$$\begin{aligned} &\max\ \mathscr{L}_s(x^\star(\lambda), \lambda) \\ &\text{s.t.} \quad \lambda \geq 0; \lambda \in \mathbb{R}^m, \end{aligned} \tag{3}$$

where $x^\star(\lambda) = arg\min_{x \in \mathbb{R}^{n-|I|}} \mathscr{L}_s(x, \lambda)$.

Problem (3) is then solved by the feasible active set method for box constrained problems proposed in [2]. Since we are considering the dual problem, it suffices to find an approximate solution, as each dual feasible solution yields a valid lower bound. We can thus prune the branch-and-bound node as soon as the current upper bound is exceeded by the value of any feasible iterate produced in a solution algorithm for the dual problem.

Experimental results for randomly generated instances show that, in the quadratic case (i.e. $p = 1$), the devised branch-and-bound method compares favorably to the MIQP solver of `CPLEX 12.6` when the number of constraints is small.