Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 55 (2016) 93–96

www.elsevier.com/locate/endm

A Branch&Price&Cut algorithm for the Vehicle Routing Problem with Intermediate Replenishment Facilities

Paolo Gianessi [1]

École des Mines de Saint-Étienne, FAYOL-EMSE, CNRS:UMR6158, LIMOS, France

Alberto Ceselli [2]

Dipartimento di Informatica, Università degli Studi di Milano, Italy

Lucas Létocart [3] Roberto Wolfler Calvo [3]

Université Paris 13, Sorbonne Paris Cité, LIPN, CNRS, (UMR 7030), France

Abstract

We present a Branch&Price&Cut algorithm for the Vehicle Routing Problem with Intermediate Replenishment Facilities that relies on a new extended formulation. The aim of this latter is to tackle symmetry issues by dropping out the vehicle index. The linear relaxation is further strengthened by adding valid inequalities.

Keywords: Column Generation, Valid Inequalities, Branch&Price&Cut, Vehicle Routing Problem with Intermediate Replenishment Facilities.

1 Introduction

The *Vehicle Routing Problem with Intermediate Replenishment Facilities (VRPIRF)* is defined on a graph where the node set consists of a *central depot* Δ, a set C of n *customers*, and f *replenishment facilities*.

[1] Email: paolo.gianessi@emse.fr

[2] Email: Alberto.Ceselli@unimi.it

[3] Email: {lucas.letocart,roberto.wolfler}@lipn.univ-paris13.fr

http://dx.doi.org/10.1016/j.endm.2016.10.024

The aim is to find a least cost set of *routes* that visits each client exactly once, the cost of a route being the sum of the costs of the visited arcs. Each client has a *demand* and can be served by one of the n_K homogeneous, fixed capacity *vehicles* based at the depot. Furthermore, vehicles can recharge at replenishment facilities so as to perform not one but a sequence of routes called a *rotation*. However, the rotation of a vehicle must start and end at the depot and its total *duration* (the sum of the travel, service and recharge times associated with the visited arcs, clients, and depots, respectively) must not exceed a given *shift length*.

VRPIRF [11] is the particular case of the *Multiple Depot VRP with Inter-Depot routes* (*MDVRPI*, [9]) with only one depot. MDVRPI itself is a generalization of the *Multi-Depot VRP (MDVRP)* in which each depot acts both as the base for the vehicles of its own fleet, and as a facility for vehicles based at other depots. Hence, VRPIRF turns out to belong to the family of Multi-Depot VRPs (see e.g. [2]), one of the most investigated families of VRPs. The multiple use of vehicles is an element that VRPIRF has also in common with the *Multi-Trip VRP (MTVRP)* [8].

In Section 2, we describe an extended formulation which makes use of *replenishment arcs* and *arrival times* together with valid connectivity inequalities, while Section 3 is devoted to the description of the Branch&Price&Cut algorithm.

2 Formulation

We propose a new Set-Partitioning formulation without the vehicle index for the VRPIRF. A solution to overcome vehicle-related symmetry issues, which affect some previous formulations, consists in using *arrival times* and *replenishment arcs*. Arrival times (inspired by e.g. [1], [6]) enable to keep track of the elapsed time along a rotation: the association between a vehicle and the routes it performs to compute its total service time can be disregarded, and the vehicle index removed. Further, arrival times assure the connection of a solution as a *side-effect*. However, in order to use them, a rotation must be represented as a sequence of arcs in which each intermediate has indegree and outdegree equal to 1. This representation shift is what *replenishment arcs* (see e.g. [6], [10]) $A_P = C \times C$ allow to do, as they model recharges in between two clients so that facility nodes are no more needed. We will use them along with *base arcs* $A_0 = V \times V$, where node set is $V = \{\Delta\} \cup C$.

As to decision variables, we have three sets of binary variables, namely *route variables* x_r, *base arc variables* x_{ij}, $ij \in A_0$ and *replenishment arc variables* w_{ij}, $ij \in A_P$, whereas *arrival time variables* z_{ij}, $i, j \in V$, are real nonnegative. Along with problem-defining contraints, we introduce *connectivity inequalities* in order to refine the fractional solution of a node of the Branch&Bound tree and tighten the lower bound. They generalize *subtour elimination constraints (SECs)* in that both