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Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 55 (2016) 139–142

www.elsevier.com/locate/endm

Intersection of Longest Paths in Graph Classes

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Abstract

Let G be a graph and $lpt(G)$ be the size of the smallest set $S \subseteq V(G)$ such that every longest path of G has at least one vertex in S. If $lpt(G) = 1$, then all longest paths of G have non-empty intersection. In this work, we prove that this holds for some graph classes, including ptolemaic graphs, P_4-sparse graphs, and starlike graphs, generalizing the existing result for split graphs.

Keywords: intersection of longest paths, graph classes, ptolemaic graphs, P_4-sparse graphs, starlike graphs

[1] This work was partially supported by CNPq, Brazil.

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http://dx.doi.org/10.1016/j.endm.2016.10.035
1571-0653/

1 Introduction

It is a well-known fact that in every simple connected graph, every two longest paths intersect, that is, they share a common vertex. In 1966, Gallai asked whether it is true that all longest paths of a connected graph share a common vertex. Even though the answer for Gallai's question is known to be negative for general graphs [4], many graph classes answer positively to this question. This is the case for split graphs [7], interval graphs [1], outerplanar graphs and 2-trees [4], circular-arc graphs [6], and graphs with matching number smaller than three [3]. In order to approach this problem, Rautenbach and Sereni [8] defined $lpt(G)$ to be the size of the smallest set $S \subseteq V(G)$ such that every longest path of G intersects S, where *lpt* stands for *longest path transversal*. Let $\mathcal{C}$ be a class of graphs. If $lpt(G) = 1$ for all $G \in \mathcal{C}$, then the answer for Gallai's question is positive in $\mathcal{C}$. In [8], the authors also provide upper bounds on $lpt(G)$ for general graphs and for some specific graph classes, such as planar and bounded treewidth graphs.

In this work, we determine graph classes that have positive answer for Gallai's question, including ptolemaic graphs, starlike graphs and P_4-sparse graphs. We also prove that if the blocks of a given graph are a split graph, an interval graph or have a universal vertex, then $lpt(G) = 1$.

2 Ptolemaic Graphs

A connected graph is a *ptolemaic graph* if for every four vertices u_1, u_2, u_3, u_4 of G, the inequality $d_{12}d_{34} \leq d_{13}d_{24} + d_{14}d_{23}$ is satisfyed, where d_{ij} is the distance between vertices u_i and u_j in G. The following characterization of ptolemaic graphs has been given by Howorka:

Theorem 2.1 (Howorka [5]) *The following conditions are equivalent:*

1. *G is a ptolemaic graph.*
2. *G is a* gem-*free graph and a chordal graph.*
3. *G is a distance-hereditary graph and a chordal graph.*
4. *For every two non-disjoint maximal cliques Q and Q' of G, $Q \cap Q'$ separates $Q \setminus Q'$ and $Q' \setminus Q$.*

A *minimal vertex separator* of G is a uv-minimal separator for some pair of vertices $u, v \in V(G)$. A family of sets $F = \{F_1, F_2, ..., F_k\}$ is said to be *laminar* if $F_i \cap F_j \neq \emptyset$ implies $F_i \subseteq F_j$ or $F_j \subseteq F_i$ $\forall i, j$. Uehara and Uno [9] proved the following theorem concerning the minimal separators of ptolemaic graphs:

graphs: