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ScienceDirect
Electronic Notes in DISCRETE MATHEMATICS
Electronic Notes in Discrete Mathematics 55 (2016) 181–184
www.elsevier.com/locate/endm

Diameter, minimum degree and hyperbolicity constant in graphs

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Abstract

In this work, we obtain good upper bounds for the diameter of any graph in terms of its minimum degree and its order, improving a classical theorem due to Erdös, Pach, Pollack and Tuza. We use these bounds in order to study hyperbolic graphs (in the Gromov sense). Since computing the hyperbolicity constant is an almost intractable problem, it is natural to try to bound it in terms of some parameters of the graph. Let $\mathcal{H}(n,\delta_0)$ be the set of graphs G with n vertices and minimum degree δ_0. We study $a(n,\delta_0) := \min\{\delta(G) \mid G \in \mathcal{H}(n,\delta_0)\}$ and $b(n,\delta_0) := \max\{\delta(G) \mid G \in$

http://dx.doi.org/10.1016/j.endm.2016.10.045

$\mathcal{H}(n, \delta_0)\}$. In particular, we obtain bounds for $b(n, \delta_0)$ and we compute the precise value of $a(n, \delta_0)$ for all values of n and δ_0.

Keywords: Diameter, minimum degree, finite graphs, Gromov hyperbolicity, hyperbolicity constant.

1 Introduction

All graphs considered in this paper are undirected, connected and simple. Let us denote by $G = (V, E)$ a graph such that every edge has length equal to 1. Here $V = V(G)$ denotes the set of vertices of G and $E = E(G)$ the set of edges of G. The degree of $v \in V(G)$ is the number of edges incident to the vertex and is denoted $\deg(v)$. The *diameter* of a graph is defined as $\operatorname{diam}(G) := \max\{d(x, y) \mid (x, y) \in G\}$, while the *diameter of the vertices* of a graph is defined as $\operatorname{diam} V(G) := \max\{d(x, y) \mid (x, y) \in V(G)\}$. The maximum and minimum degree of a graph G are $\Delta := \max\{\deg(v) \mid v \in V(G)\}$, $\delta_0 := \min\{\deg(v) \mid v \in V(G)\}$.

In the design of communication networks, it is common to take into account limitations on the vertex degrees and the diameter. Throughout the years, problems related with the diameter and degree have attracted the attention of many researchers and they have numerous applications (see [5] for an overview on results related to this topic).

In this paper, we focus on improving a result due to Erdös, Pach, Pollack and Tuza (see [2]), which gives an asymptotically sharp upper bound for the diameter of a connected graph in terms of its minimum degree and its order.

On the other hand, on the second part of this work, we deal with hyperbolic graphs in the Gromov sense.

Gromov hyperbolicity was introduced by Mikhail Leonidovich Gromov in the setting of geometric group theory, but has played an increasing role in analysis on general metric, with applications to the Martin boundary, invariant metrics in several complex variables and extendability of Lipschitz mappings. The concept of hyperbolicity appears also in discrete mathematics, algorithms and networking. Another important application of these spaces is the secure

[1] Supported in part by a grant from Ministerio de Economía y Competitividad (MTM 2013-46374-P), Spain.
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