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Vertices, edges, distances and metric dimension in graphs

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Abstract

Given a connected graph $G = (V, E)$, a set of vertices $S \subset V$ is an edge metric generator for G, if any two edges of G are identified by S by mean of distances to the vertices in S. Moreover, in a natural way, S is a mixed metric generator, if any two elements of G (vertices or edges) are identified by S by mean of distances. In this work we study the (edge and mixed) metric dimension of graphs.

Keywords: mixed metric dimension, edge metric dimension, metric dimension.

Parameters related to distances in graphs have attracted the attention of several researchers since several years, and recently, one of them has centered several investigations, namely, the metric dimension. A vertex v of a connected graph G *distinguishes* two vertices u, w if $d(u, v) \neq d(w, v)$, where $d(x, y)$ represents the length of a shortest $x - y$ path in G. A subset of vertices S of G is a *metric generator* for G, if any pair of vertices of G is distinguished by at least one vertex of S. The minimum cardinality of any metric generator for G is the *metric dimension* of G. This concept was introduced by Slater in [5] in connection with some location problems in graphs. On the other hand,

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the concept of metric dimension was independently introduced by Harary and Melter in [2].

One can now consider the following situation. A metric generator uniquely recognizes the vertices of a graph in order to look out how they "behave". However, what does it happen if there are anomalous situations occurring in some connections (edges) between some vertices? Is it possible that metric generators would properly identify the edges in order to also see their behaving? The answer to this question is negative. In connection with this, the following concepts deserve to be considered.

Given a connected graph $G = (V, E)$, a vertex $v \in V$ and an edge $e = uw \in E$, the distance between the vertex v and the edge e is defined as $d_G(e, v) = \min\{d_G(u, v), d_G(w, v)\}$. A vertex $w \in V$ *distinguishes* two edges $e_1, e_2 \in E$ if $d_G(w, e_1) \neq d_G(w, e_2)$. A set $S \subset V$ is an *edge metric generator* for G if any two edges of G are distinguished by some vertex of S. The smallest cardinality of an edge metric generator for G is the *edge metric dimension* and is denoted by $edim(G)$ [3]. Moreover, a kind of mixed version of these two parameters described above is of interest. That is, a vertex v of G *distinguishes* two elements (vertices or edges) x, y of G if $d_G(x, v) \neq d_G(y, v)$. Now, a set $S \subset V$ is a *mixed metric generator* if any two elements of G are distinguished by some vertex of S. The smallest cardinality of a mixed metric generator for G is the *mixed metric dimension* and is denoted by $mdim(G)$ [4].

1 Results

As stated, there are several graphs in which no metric generator is also an edge metric generator. In this sense, one could think that probably any edge metric generator S is also a standard metric generator. Nevertheless, this is again further away from the reality, although there are several graph families in which such facts occur. In [3], among other results, some comparison between these two parameters above were discussed. In contrast with this, for the case of mixed metric dimension, it clearly follows that that any mixed metric generator is also a metric generator and an edge metric generator. In this sense, the following relationship immediately follows. For any graph G, $mdim(G) \geq \max\{dim(G), edim(G)\}$. From now on, we present several results concerning the (edge, mixed) metric dimension of graphs. First of all, we remark the next complexity result.

Theorem 1.1 [3] *Computing the edge metric dimension of graphs is NP-hard.*

The result above was proved by using a reduction from the 3-SAT problem.