Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 55 (2016) 199–202 www.elsevier.com/locate/endm

On the shelling antimatroids of split graphs

Keno Merckx [1]

Département d'Informatique
Université Libre de Bruxelles
Brussels, Belgium

Jean Cardinal

Département d'Informatique
Université Libre de Bruxelles
Brussels, Belgium

Jean-Paul Doignon

Département d'Informatique
Université Libre de Bruxelles
Brussels, Belgium

Abstract

Unlike poset antimatroids, chordal graph shelling antimatroids have received little attention as regard their structures, optimization properties and associated circuits. Here we consider a special case of those antimatroids, namely the split graph shelling antimatroids. We establish a connection between the structure of split graph shelling antimatroids and poset shelling antimatroids. We discuss some applications of this new connection, in particular, we give a simple polynomial time algorithm to find a maximum weight feasible set in split graph shelling antimatroids.

Keywords: Antimatroid, split graph, shelling, poset.

http://dx.doi.org/10.1016/j.endm.2016.10.049
1571-0653/

1 Introduction

Many classical problems in combinatorial optimization have the following form.

Problem 1.1 *For a set system $(V, \mathcal{F})$ and for a function $w : V \to \mathbb{R}$, find a set F of $\mathcal{F}$ maximizing the value of*

$$w(F) = \sum_{f \in F} w(f).$$

For instance, the problem is known to be efficiently solvable for the independent sets of matroids using the greedy algorithm. Since antimatroids capture a combinatorial abstraction of convexity in the same way as matroids capture linear dependence, we investigate the optimization of linear objective functions for antimatroids.

We recall that a set system $(V, \mathcal{F})$, where V is a finite set of elements and $\varnothing \neq \mathcal{F} \subseteq 2^V$, is an *antimatroid* when

$$V \in \mathcal{F}, \tag{AM0}$$
$$\forall F_1, F_2 \in \mathcal{F} \Rightarrow F_1 \cup F_2 \in \mathcal{F}, \tag{AM1}$$
$$\forall F \in \mathcal{F} \text{ and } F \neq \varnothing \Rightarrow \exists\, f \in F \text{ such that } F \setminus \{f\} \in \mathcal{F}. \tag{AM2}$$

The *feasible sets* of the antimatroid $(V, \mathcal{F})$ are the members of $\mathcal{F}$. We call *path* any feasible set that cannot be decomposed into the union of two other (non-empty) feasible sets.

Antimatroids arise naturally from various kinds of shellings and searches on combinatorial objects, and appear in various contexts in mathematics and computer science. Dilworth [4] first examined structures very close to antimatroids in terms of lattice theory. Later, Edelman [5] and Jamison [7] studied the convex aspects of antimatroids. Korte, Lovász and Schrader [8] considered antimatroids as a subclass of greedoids. Today, the concept of antimatroid appears in many fields of mathematics such as formal language theory (Boyd and Faigle [2]), choice theory (Koshevoy [9]), game theory (Algaba *et al.* [1]) and mathematical psychology (Falmagne and Doignon [6]) among others. The concept of a convex geometry is dual to the one of an antimatroid.

For instance, one particular class of antimatroids comes from shelling processes over posets by removing successively the maximum elements. Let $(V, \leq)$

[1] Email: `kmerckx@ulb.ac.be`