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A Polynomial Recognition of Unit Forms

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Abstract

In this paper we introduce a polynomial algorithm for the recognition of weakly nonnegative unit forms. The algorithm identify hypercritical restrictions testing every 9-point subset of the quadratic form associated graph. With Depth First Search strategy, we use a similar approach for the weakly positive recognition.

Keywords: unit form, polynomial algorithm, graph. MSC: 11E04, 16G60, 42A82.

1 Introduction

An integral quadratic form q is defined as $q(x) = \sum_{i \leq j} a_{ij} x_i x_j$, for $x \in \mathbb{Z}^n$. In this paper, we are interested in **unit form**, that is an integral quadratic form

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where $a_{ii} = 1$, for all i. Its corresponding symmetric bilinear form is such that $q(x) = \frac{1}{2}q(x,x)$. The recognition of weakly nonnegative and weakly positive unit forms have an important role in the representation theory of algebras, however, some concepts related to computational complexity theory have not been fully described.

Dean and De La Peña [1] developed an algorithm to decide whether a given unit form is weakly nonnegative. The strategy was to generate all positive roots and make some tests to identify whether the unit form is weakly nonnegative. Despite being a significant development, the weakly nonnegative unit forms can have an infinite number of positive roots, and this strategy become unfeasible.

A very interesting solution for weakly nonnegative recognition comes from the hypercritical unit forms, classified by Unger [5]. All unit forms in the Unger's list have 9 or less vertices. Therefore, we use an algorithm to test all 9-point subsets. That strategy give us a polynomial algorithm of complexity $O(n^9)$. By adding Depth First Search approach, we use a similar strategy in the weakly positive recognition.

2 Basic Concepts

A vector $x \in \mathbb{Z}^n$ is said to be positive, written $x > 0$, provided $x \neq 0$ and $x_i \geq 0$ for all i. A unit form q is **weakly positive** if $q(x) > 0$ for all positive $x \in \mathbb{Z}^n$, or **weakly nonnegative**, if $q(x) \geq 0$ for all positive $x \in \mathbb{Z}^n$.

Definition 2.1 [2] A unit form q is said to be critical, resp. hypercritical, if every proper restriction q' is weakly positive, resp. weakly nonnegative, but q itself is not.

All critical forms were classified by von Höhne [4] and all hypercritical by Unger [5]. A unit form is properly represented by quivers.[5] A **Quiver** $Q = (Q_0, Q_1)$ is a finite and connected graph with a set of vertices $Q_0 = \{1, \ldots, n\}$ and a set of edges Q_1, with possibly multiple edges but without loops.

3 A Polynomial Approach

The following corollary gives support to the polynomial algorithm for weakly nonnegative recognition.

[5] See more details about quiver representations in [2].