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Drawing graphs with mathematical programming and variable neighborhood search

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Abstract

In the Graph Drawing problem a set of symbols must be placed in a plane and their connections routed. To produce clear, easy to read diagrams, this problem is usually solved trying to minimize edges crossing and the area occupied, resulting in a NP-Hard problem. Our research focuses in drawing Entity Relationship (ER) diagrams, a challenging version of the problem where nodes have different sizes. Mathematical Programming models for the two solution phases, node placement and connection routing, are discussed and their exact resolution by an Integer Programming (IP) solver is evaluated. As the first phase proved to be specially hard to be solved exactly, a hybrid Variable Neighborhood Search (VNS) heuristic is proposed. Using IP techniques we obtained provably optimal (or close to optimal) solutions for the two different phases, at the expense of a large computational effort. We also show that our VNS based heuristic approach can produce close to optimal solutions in very short times for the hardest part of the solution process. Using either methods we have produced clearly better drawings than existing solutions.

Keywords: Grid graph drawing, mixed integer programming, variable neighborhood search.

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1 Introduction

In the Graph Drawing problem a set of symbols must be placed in a plane and their connections routed. The objective is to produce aesthetically pleasant, easy to read diagrams. As a primary concern one usually tries to minimize edges crossing, edges' length, waste of space and number of bents in the connections. When formulated with these constraints the problem becomes NP-Hard[4]. In practice many additional complicating requirements can be included, such as non-uniform sizes for symbols[3]. Thus, some heuristics such as the generalized force-direct method and Simulated Annealing [2] have been proposed to tackle this problem. [1] uses a grid structure to approach the Entity-Relationship (ER) drawing problem, emphasizing the differences between ER drawing and the more classical circuit drawing problems. [6] presented different ways of producing graph layouts (e.g., tree, orthogonal, visibility representations, hierarchic, among others) for general graphs with applications on different subjects.

The ability to automatically produce high quality layouts is very important in many applications, one of these is Software Engineering: the availability of easy to understand ER diagrams, for instance, can improve the time needed for developers to master database models and increase their productivity. In this work we present Integer Programming and Local Search approaches to solve the Entity-Relationship graph drawing problem. To the best of our knowledge this is the first time that Integer Programming is used in a comprehensive graph drawing problem. Our solution approach involves two phases: (i) firstly the optimal placement of entities is solved, i.e.: entities are positioned so as to minimize the distances between connected entities; and (ii) secondly, edges are routed minimizing bends and avoiding the inclusion of connectors too close. We observed that the model of the first phase was significantly harder to solve exactly. Thus, a heuristic based in Variable Neighborhood Search (VNS)[5] is proposed. Our approach allowed us to determine lower and upper bounds for instances generated from real world applications, leading to provably optimal (or very close to optimal) solutions for phases (i) and (ii). This paper is organized as follows: Section 2 briefly presents the problem, Sections 3 and 4 describe the Integer Programming formulations used in this work for the first and second phases respectively, Section 5 presents the proposed VNS heuristic and finally, Section 6 presents some computational results and conclusions.

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