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Completeness of the 95256-cap in $PG(12, 4)$

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Abstract

We describe an algorithm for testing the completeness of caps in PG(r, q), q even. It allowed us to check that the 95256-cap in PG$(12, 4)$ recently found by Fu el al. (see [3]) is complete.

Keywords: Projective spaces, caps, complete caps.

1 Introduction

Let PG(r, q) be the r-dimensional projective space over the Galois field $\mathbb{F}_q$. An n-cap in PG(r, q) is a set of points no three of which are collinear. An n-cap in PG(r, q) is called complete if it is not contained in an $(n + 1)$-cap in PG(r, q); see [4].

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The points of a complete n-cap in $\mathrm{PG}(r-1,q)$ can be treated as columns of a parity check matrix of an $[n,n-r,4]_q$ linear code with the exceptions of the complete 5-cap in $\mathrm{PG}(3,2)$ and the complete 11-cap in $\mathrm{PG}(4,3)$ corresponding to the binary $[5,1,5]_2$ code and to the Golay $[11,6,5]_3$ code respectively.

An n-cap in $\mathrm{PG}(r,q)$ of maximal size is called a maximal cap in $\mathrm{PG}(r,q)$. A classical problem on caps is to determine the maximal size of complete caps in $\mathrm{PG}(r,q)$. This is also known as the packing problem; see [5]. Denote the size of a maximal cap in $\mathrm{PG}(r,q)$ as $m_2(r,q)$, and the largest size of a known complete cap as $\overline{m}_2(r,q)$.

Of particular interest is the case $q=4$, due the connection with quantum error correction established in [2], where a class of quantum codes, the quantum stabilizer codes, is described in terms of certain additive quaternary codes.

The value of $m_2(r,4)$ is known for $k \leq 4$: $m_2(2,4)=6$, $m_2(3,4)=17$, and $m_2(4,4)=41$; see [1].

In [3] it is proved that $\overline{m}_2(8,4)=2136$, $\overline{m}_2(9,4)=5124$, $\overline{m}_2(10,4)=15840$, $\overline{m}_2(11,4)=36084$ and a 95256-cap in $PG(12,4)$ is also given.

Their results have been obtained by computer-supported recursive constructions. They also present an algorithm for checking completeness of a cap. This algorithm allowed checking the completeness of the caps for $k \leq 11$, but it is too computationally expensive for the case $k=12$. As they wrote: "But as for checking completeness of larger caps in $PG(r,4)$, $r \geq 12$, new algorithms are needed."; see [3, Section 5]. We propose a new fast algorithm that allowed to face also this case: we verified that the 95256-cap in $PG(12,4)$ is complete, so $\overline{m}_2(12,4)=95256$. Our algorithm is based on a compact representation of the points of $\mathrm{PG}(r,q)$, q even, and on minimizing the computational costs of the operations more often performed during the check of the completeness of the cap.

Section 2 describes the algorithm and applies it in $PG(12,4)$. Section 3 contains the generalization of the algorithm to other even values of q and other dimensions.

2 A new algorithm for checking completeness of a cap

In [3] an algorithm for checking completeness of a cap $\mathscr{C}$ in $PG(r,4)$ is presented. It is based on a bijective map ϕ between points in $PG(r,4)$ and a subset $T(r)$ of the positive integer set $\mathbb{N}$. Let $P \in PG(r,4)$, $P=(x_0,x_1,...,x_r)^T$; then $\phi(P)=4^r x_0+4^{r-1}x_1+\cdots+4x^{r-1}+x^r$.