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On the Sharpness of the Griesmer Bound

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Abstract

We investigate the following version of the main problem of coding theory: Given the integer k and the prime power q, what is the value of

$$t_q(k) := \max_d n_q(k,d) - g_q(k,d).$$

We give several formulations of this problem: in terms of linear codes, arcs and minihypers. We provide general constructions that give upper bounds on $t_q(k)$.

Keywords: linear codes, Griesmer bound, maximal arcs in finite projective planes, optimal codes

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The Griesmer bound for linear $[n, k, d]_q$-codes is a lower bound on the length n as a function of q, k, and d [4,7]:

$$n \geq g_q(k, d) := \sum_{i=0}^{k-1} \lceil \frac{d}{q^i} \rceil.$$

It is known that for fixed q and k Griesmer codes do exist for all sufficiently large d [2,5]. In fact, it follows from the theoreo of Belov-Logachev-Sandimirov [1] that this is true for all $d \geq (k-2)q^{k-1}+1$ On the other hand, a less known result by Dodunekov [2] says that for fixed q and d and $k \to \infty$

$$n_q(k, d) - g_q(k, d) \to \infty.$$

The following question can be viewed as a version of the main problem of coding theory:

Problem A. Given the integer k and the prime power q, what is the exact value of

$$t_q(k) := \max_d n_q(k, d) - g_q(k, d),$$

or, in other words, given k and q, what is the smallest value of t, such that there exists a $[t + g_q(k, d), k, d]_q$-code.

It is well-known that $t_q(2) = 0$, i.e. two-dimensional Griesmer codes exist for all q and all d (see e.g. [5]). For values of $k \geq 3$ the problem is unsolved. In the case of $k = 3$ it was asked by S. Ball in the following way:

For a fixed $n-d$, is there always a 3-dimensional code meeting the Griesmer bound (maybe a constant or $\log q$ away)?

From the known tables of the best arcs and linear codes, we can find exact values and estimates for $t_q(k)$ for small q and k. We have $t_q(3) = 1$ for $q \leq 19$ and $t_q(3) \leq 2$ for all $23 \leq q \leq 29$. For larger dimensions we know that: $t_3(4) = 1, t_4(4) = 1, t_5(4) = 2$ (with $t = 2$ only for $d = 25$), $t(5(5) \leq 5$. Now we are going to state this problem geometrically.

Let us write the minimum distance d as

$$d = sq^{k-1} - \lambda_{k-2}q^{k-2} - \ldots - \lambda_1 q - \lambda_0, \tag{1}$$

where $0 \leq \lambda_i < q$. It is easily checked that

$$g_q(k, d) = sv_k - \lambda_{k-2}v_{k-1} - \ldots - \lambda_1 v_2 - \lambda_0 v_1, \tag{2}$$

where $v_i = (v^i - 1)/(v - 1)$. Now a Griesmer $[n, k, d]_q$-code can be associated